Day 35 **A Tier 10-a-day: Model Solutions**

1 Sharing in a ratio

Hugo and Maya share £180 in the ratio 5 : 7.
How much more money does Maya get than Hugo?

Total number of parts = 5 + 7 = 12
One part = £180 ÷ 12 = £15
Maya gets 7 × £15 = £105 and Hugo gets 5 × £15 = £75
Difference = £105 − £75 = £30
Answer: £30

2 Surface area of a prism

The diagram shows a triangular prism.
The cross-section is a right-angled triangle.
Work out the total surface area of the prism.
The diagram is not drawn accurately.

Hypotenuse = $\sqrt{24^2 + 7^2}$ = 25 cm
Two triangular ends: $2 \times \frac{1}{2} \times 24 \times 7 = 168$
Three rectangles: (24 + 7 + 25) × 10 = 560
Total = 168 + 560 = 728
Answer: 728 cm^2

3 Expanding double brackets

Expand and simplify
$(3x - 5)^2$

Multiply every term in the first bracket by every term in the second
$= (3x - 5)(3x - 5) = 9x^2 - 15x - 15x + 25$
$= 9x^2 - 30x + 25$
Answer: $9x^2 - 30x + 25$

4 HCF and LCM in index form

A = $2^3 \times 5^3$
B = $2 \times 3 \times 5^2$
Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.
Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): 2×5^2
LCM: for each prime take the higher power: $2^3 \times 3 \times 5^3$
Answer: HCF = 2×5^2, LCM = $2^3 \times 3 \times 5^3$

5 Reading a histogram

The histogram shows information about the length of some items.
(a) Work out the number of items with a length in the class 10 to 30 mm.
(b) Work out an estimate for the number with a length greater than 50 mm.

(a) Frequency = frequency density × class width
2.4 × 20 = 48
(b) 50 to 60: 2 × 10 = 20
Class 60 to 70: 1.8 × 10 = 18
Total = 20 + 18 = 38
Answer: (a) 48 (b) 38

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6 Successive percentage changes

The number of followers of a channel is increased by 10% and then the new amount is decreased by 20%.
Work out the overall percentage decrease.

Multipliers: increase of 10% is 1.1, decrease of 20% is 0.8
Overall multiplier = 1.1 × 0.8 = 0.88
0.88 means a percentage decrease of 12%
Answer: 12% decrease

7 Changing a ratio

A bag contains red counters and blue counters in the ratio 7 : 1.
22 more blue counters are added to the bag.
The ratio of red counters to blue counters is now 7 : 3.
How many counters were in the bag at the start?

Let red = 7x and blue = 1x. After adding: 7x : (1x + 22) = 7 : 3
$\frac{7x}{1x + 22} = \frac{7}{3}$, so 3 × 7x = 7(1x + 22)
21x = 7x + 154, so 14x = 154, x = 11
Start: 7x + 1x = 8 × 11 = 88 counters
Answer: 88

8 Expanding three brackets

Expand and simplify
(x − 1)(x + 3)(x + 4)

Expand the first two brackets: (x − 1)(x + 3) = x^2 + 2x − 3
Multiply by the third bracket: (x^2 + 2x − 3)(x + 4)
= x^3 + 2x^2 − 3x + 4x^2 + 8x − 12
= x^3 + 6x^2 + 5x − 12
Answer: x^3 + 6x^2 + 5x − 12

9 Reverse percentages

In a sale, all prices are reduced by 6%. (Calculator)
A watch costs £408.90 in the sale.
Work out the original price of the watch.

The sale price is 94% of the original price.
94% = £408.90, so 1% = £408.90 ÷ 94 = £4.35
100% = £4.35 × 100 = £435
Answer: £435

10 Bearings with the cosine rule

Ship A sails 22 km from a port on a bearing of 320°.
Ship B sails 7 km from the same port on a bearing of 285°. (Calculator)
Work out the distance between the two ships.
Give your answer to 3 significant figures.

The angle between the two paths = |320° − 285°| = 35°
$d^2 = 22^2 + 7^2 - 2 \times 22 \times 7 \times \cos 35°$
d^2 = 280.70
d = 16.8
Answer: 16.8 km