Day 345 **A Tier 10-a-day: Model Solutions**

1 Circle area and circumference

A circular pond has radius 13 m.
Work out, in terms of π,
(a) the circumference,
(b) the area.

(a) Circumference = $\pi d = \pi \times 26 = 26\pi$
(b) Area = $\pi r^2 = \pi \times 13^2 = 169\pi$
Answer: (a) 26π m (b) 169π m^2

2 Median and interquartile range

Here are the scores of 11 students in a test.
32 55 48 57 50 27 44 15 49 40 56
(a) Work out the median.
(b) Work out the interquartile range.

Put in order: 15, 27, 32, 40, 44, 48, 49, 50, 55, 56, 57
(a) The middle value is number 6, so the median is 48
(b) Lower half: 15, 27, 32, 40, 44, so LQ = 32
Upper half: 49, 50, 55, 56, 57, so UQ = 55
IQR = 55 − 32 = 23
Answer: (a) 48 (b) 23

3 Difference of two squares

(a) Factorise $16y^2 - 25z^2$
(b) Use the same idea to work out $44^2 - 6^2$ without a calculator.

(a) $16y^2 - 25z^2 = (4y)^2 - (5z)^2$
$(4y - 5z)(4y + 5z)$
(b) $a^2 - b^2 = (a + b)(a - b)$, so $44^2 - 6^2 = (44 + 6)(44 - 6)$
$= 50 \times 38 = 1900$
Answer: (a) (4y − 5z)(4y + 5z) (b) 1900

4 Interior angles of polygons

The size of each angle of a quadrilateral is shown as an expression:
(2x + 20)°, (x + 10)°, (3x − 30)°, (3x − 18)°
Work out the value of x.

Sum of angles = (4 − 2) × 180 = 360°
2x + 20 + x + 10 + 3x − 30 + 3x − 18 = 360
9x − 18 = 360
9x = 378
x = 42
Answer: x = 42

5 Reverse percentages

Noah's weekly pay is increased by 5%.
Her new weekly pay is £315.
Work out her weekly pay before the increase.

105% represents £315
1% = £315 ÷ 105 = £3
100% = £3 × 100 = £300.00
Answer: £300.00

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6 Terminating and recurring decimals

Here are four fractions: $\frac{7}{15}$ $\frac{3}{6}$ $\frac{9}{30}$ $\frac{4}{5}$

Which one of these fractions is equal to a recurring decimal?

Simplify each fraction: $\frac{7}{15}$, $\frac{3}{6} = \frac{1}{2}$, $\frac{9}{30} = \frac{3}{10}$, $\frac{4}{5}$

A fraction in simplest form is a terminating decimal only if its denominator has no prime factors other than 2 and 5.

$\frac{7}{15}$ has denominator $15 = 3 \times 5$, which includes a factor other than 2 and 5.

Answer: $\mathbf{\frac{7}{15}}$

7 Perimeter and area with algebra

An isosceles triangle has two equal sides of length $(2x + 8)$ cm and $(3x + 1)$ cm.
The third side is 14 cm.
Work out the perimeter of the triangle.

The two equal sides give an equation

$2x + 8 = 3x + 1$

$x = 7$

$x = 7$

Equal sides $= 2 \times 7 + 8 = 22$ cm

Perimeter $= 22 + 22 + 14 = 58$

Answer: 58 cm

8 Fractions of an amount

Leah spends $\frac{1}{5}$ of the money on a phone case and $\frac{1}{3}$ of the money that is left on headphones.

Leah has £304 left.
How much money did Leah have to begin with?

After the second purchase $\frac{2}{3}$ of what was left after the first purchase remains.

£304 = $\frac{2}{3}$ of the amount left, so $\frac{1}{3}$ of it is £304 ÷ 2 = £152

So the amount left after the first purchase is £152 × 3 = £456

This is $\frac{4}{5}$ of the original amount, so the original amount is £456 ÷ 4 × 5 = £570

Answer: £570

9 Perpendicular bisector

A is the point $(2, -3)$ and B is the point $(8, 1)$.
Find the equation of the perpendicular bisector of AB.

Midpoint of AB $= \left(\frac{2+8}{2}, \frac{(-3)+1}{2}\right) = (5, -1)$

Gradient of AB $= \frac{4}{6} = \frac{2}{3}$

Perpendicular gradient $= -1 \div \frac{2}{3} = -\frac{3}{2}$

$y = -\frac{3}{2}x + c$ through $(5, -1)$: $-1 = -\frac{3}{2} \times 5 + c$, so $c = \frac{13}{2}$

$y = -\frac{3}{2}x + \frac{13}{2}$

Answer: $\mathbf{y = -\frac{3}{2}x + \frac{13}{2}}$

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10 Successive percentage changes

The value of a painting is decreased by 30%.
The new value is then increased by 5%.
Work out the overall percentage change.

First change: multiplier 0.7. Second change: multiplier 1.05
Overall multiplier = 0.7 × 1.05 = 0.735
0.735 − 1 = −0.265, so the overall change is 26.5%
Answer: 26.5% decrease