Day 344 **A Tier 10-a-day: Model Solutions**

1 Map scales

The scale of a map is 1 : 200000.
A road is 4 km long.
Work out the length of the road on the map, in centimetres.

4 km = 400000 cm
Length on map = 400000 ÷ 200000 = 2 cm
Answer: 2 cm

2 Loci and constructions

A point S moves so that it is always 7 cm from a fixed point X.
Describe the locus of S.

All points the same distance from X form a circle
Centre X, radius 7 cm
Answer: A circle with centre X and radius 7 cm

3 Reverse percentages

A pair of trainers costs £234 including VAT at 20%. (Calculator)
Work out the price of the pair of trainers before VAT is added.

Price including VAT is 120% of the price before VAT.
120% = £234, so 1% = £234 ÷ 120 = £1.95
100% = £1.95 × 100 = £195
Answer: £195

4 Median and interquartile range

Here are the times, in seconds, taken by 11 runners.
40 60 46 12 35 75 37 62 31 27 24
(a) Work out the median.
(b) Work out the interquartile range.

Put in order: 12, 24, 27, 31, 35, 37, 40, 46, 60, 62, 75
(a) The middle value is number 6, so the median is 37
(b) Lower half: 12, 24, 27, 31, 35, so LQ = 27
Upper half: 40, 46, 60, 62, 75, so UQ = 60
IQR = 60 − 27 = 33
Answer: (a) 37 (b) 33

5 Factorising

Factorise $2x^2 + 13x + 6$

Multiply 2 × 6 = 12. Find two numbers that multiply to 12 and add to 13: 12 and 1
$2x^2 + 13x + 6 = 2x^2 + 12x + x + 6$
Factorise in pairs to get (2x + 1)(x + 6)
Answer: (2x + 1)(x + 6)

6 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 54°.
Work out the size of angle x (angle AOB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
The angles in quadrilateral OAPB add up to 360°
x = 360 − 90 − 90 − 54 = 126
Answer: x = 126°

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7 Probability with three selections

A bag contains 7 red counters and 3 blue counters.
Three counters are taken at random without replacement.
Work out the probability that exactly one of the three counters is blue.

One order, for example blue, red, red: $\frac{3}{10} \times \frac{7}{9} \times \frac{6}{8} = \frac{7}{40}$

There are 3 possible orders, each with the same probability

$3 \times \frac{7}{40} = \frac{21}{40}$

Answer: $\mathbf{\frac{21}{40}}$

8 Perpendicular bisector

A is the point (4, 3) and B is the point (2, 5).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $(\frac{4+2}{2}, \frac{3+5}{2}) = (3, 4)$

Gradient of AB = $\frac{2}{-2} = -1$

Perpendicular gradient = $-1 \div (-1) = 1$
$y = x + c$ through (3, 4): $4 = 1 \times 3 + c$, so $c = 1$
$y = x + 1$
Answer: $y = x + 1$

9 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{3x + 6}{-x + 4}$

Multiply both sides by $(-x + 4)$: $y(-x + 4) = 3x + 6$
Expand the bracket: $-xy + 4y = 3x + 6$
Collect the x terms on one side: $-xy - 3x = -4y + 6$
Factorise: $x(-y - 3) = -4y + 6$
Divide by $(-y - 3)$: $x = \frac{-4y + 6}{-y - 3}$

Multiply the top and bottom by -1: $x = \frac{4y - 6}{y + 3}$

Answer: $\mathbf{x = \frac{4y - 6}{y + 3}}$

10 Percentages of percentages

There are 160 passengers on a plane.
25% of the passengers are children.
25% of the children are under 5 years old.
How many passengers are children under 5?

Children = 25% of 160 = 40
Under 5 = 25% of 40 = 10
Answer: 10