Day 342 **A Tier 10-a-day: Model Solutions**

1 Probabilities adding to 1

The table shows the probabilities of the outcomes of a biased spinner.
(a) Work out the value of p.
(b) The spinner is spun 300 times. Estimate the number of times it lands on 2.

(a) Probabilities add to 1: 0.05 + p + 2p + 0.35 = 1
0.4 + 3p = 1, so 3p = 0.6
p = 0.2
(b) P(2) = 0.2, so expected number = 0.2 × 300 = 60
Answer: (a) p = 0.2 (b) 60

2 Double inequalities

(a) Solve $-5 \leq 2x - 7 \leq 1$
(b) Write down all the integer values of x that satisfy this inequality.

(a) Add 7 to every part: $2 \leq 2x \leq 8$
Divide every part by 2: $1 \leq x \leq 4$
(b) The integers are 1, 2, 3, 4
Answer: (a) $1 \leq x \leq 4$ (b) 1, 2, 3, 4

3 Converting units

(a) Change 7 cm^2 to mm^2.
(b) Change 15 m^2 to mm^2.

(a) 7 cm^2 × 100 = 700 mm^2
(b) 15 m^2 × 1,000,000 = 15,000,000 mm^2
Answer: (a) 700 mm^2 (b) 15,000,000 mm^2

4 Expanding double brackets

Expand and simplify
(2x + 5)(2x + 3)

Multiply every term in the first bracket by every term in the second
= $4x^2 + 6x + 10x + 15$
= $4x^2 + 16x + 15$
Answer: $4x^2 + 16x + 15$

5 Perimeter and area with algebra

A rectangle has length (3x + 8) cm and width (3x + 6) cm.
The perimeter of the rectangle is 64 cm.
(a) Work out the value of x.
(b) Work out the area of the rectangle.

(a) 2 × [(3x + 8) + (3x + 6)] = 64
(3x + 8) + (3x + 6) = 32
6x + 14 = 32
6x = 18
x = 3
(b) Length = 17 cm and width = 15 cm
Area = 17 × 15 = 255
Answer: (a) x = 3 (b) 255 cm^2

6 Expanding with surds

Expand and simplify $2\sqrt{10}(5 + 4\sqrt{10})$
Write your answer in the form $a + b\sqrt{c}$

$2\sqrt{10} \times 5 = 10\sqrt{10}$ and $2\sqrt{10} \times 4\sqrt{10} = 8 \times 10 = 80$
$= 80 + 10\sqrt{10}$
Answer: $80 + 10\sqrt{10}$

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7 Enlargements and coordinates

Triangle LMN has vertices L(1, 2), M(4, −1), N(0, 1).
The triangle is enlarged with scale factor 4 and centre (0, 0).
Write down the coordinates of the vertices of the image.

Each vertex moves to: centre + scale factor × (vertex − centre)
L: (1, 2) → (4, 8)
M: (4, −1) → (16, −4)
N: (0, 1) → (0, 4)
Answer: L'(4, 8), M'(16, −4), N'(0, 4)

8 Quadratic inequalities

Solve $x^2 > 100$

$x^2 = 100$ at $x = \pm 10$
The curve $y = x^2$ is above 100 outside the roots
$x < -10$ or $x > 10$
Answer: $x < -10$ or $x > 10$

9 Direct proportion formulae

y is directly proportional to the square of x.
When x = 2, y = 36.
(a) Find a formula for y in terms of x.
(b) Work out the positive value of x when y = 225.

(a) $y = kx^2$. Substitute: $36 = k \times 4$, so $k = 9$
$y = 9x^2$
(b) $225 = 9x^2$, so $x^2 = 225 \div 9 = 25$, so $x = 5$
Answer: (a) $y = 9x^2$ (b) $x = 5$

10 Volume of composite solids

A solid is made from a cylinder of radius 9 cm and a hemisphere on top, with the same radius.
The total volume of the solid is 1539π cm^3.
Work out the height of the cylinder.

Hemisphere = $\frac{2}{3} \times \pi \times 9^3 = 486\pi$
Cylinder volume = $1539\pi - 486\pi = 1053\pi$
$\pi \times 9^2 \times h = 1053\pi$
$h = 1053 \div 81 = 13$
Answer: 13 cm