Day 339 **A Tier 10-a-day: Model Solutions**

1 Loci and constructions

PQ is a straight line.
A point P moves so that it is always 8 cm from the line PQ, on one side of it.
Describe the locus of P.

Points a fixed distance from a straight line, on one side, form a parallel line
It is parallel to PQ and 8 cm from it
Answer: A straight line parallel to PQ, 8 cm away from it

2 Successive percentage changes

The population of a village is increased by 50%.
The new population is then decreased by 20%.
Work out the overall percentage change.

First change: multiplier 1.5. Second change: multiplier 0.8
Overall multiplier = 1.5 × 0.8 = 1.2
1.2 − 1 = 0.2, so the overall change is 20%
Answer: 20% increase

3 Standard form calculations

Work out $(8 \times 10^{-4}) \times (2.5 \times 10^{-4})$
Give your answer in standard form.

Multiply the numbers and add the powers of 10: $8 \times 2.5 = 20$ and $10^{-4} \times 10^{-4} = 10^{-8}$
20×10^{-8}
20 is not between 1 and 10, so $= 2 \times 10^{-7}$
Answer: 2×10^{-7}

4 Best buys

A shop sells flour in 3 different sizes.
Small bag: 200 g for £1.85
Medium bag: 400 g for £3.45
Large bag: 1000 g for £7.30
Which bag is the best value for money? You must show your working.

Small bag: 185p ÷ 2 = 92.5p per 100 g
Medium bag: 345p ÷ 4 = 86.25p per 100 g
Large bag: 730p ÷ 10 = 73p per 100 g
The large bag has the lowest price per 100 g, so it is the best value
Answer: The large bag

5 Quadratic graph from its roots

The diagram shows the graph of $y = -x^2 + bx + c$.
Find the value of b and the value of c.

The curve crosses the x-axis at $x = -3$ and $x = -1$
So $y = -(x + 3)(x + 1)$
Expand: $y = -x^2 - 4x - 3$
$b = -4$ and $c = -3$
Answer: $b = -4$, $c = -3$

6 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 68°.
Work out the size of angle x (angle AOB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
The angles in quadrilateral OAPB add up to 360°
$x = 360 - 90 - 90 - 68 = 112$
Answer: $x = 112°$

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7 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + 5x - 30 = 0$ can be rearranged to give $x = \frac{30 - x^3}{5}$

(b) Show that the equation has a solution between x = 2 and x = 3

(a) Add 30 and subtract 5x from both sides: $5x = 30 - x^3$

Divide both sides by 5: $x = \frac{30 - x^3}{5}$

(b) Let $f(x) = x^3 + 5x - 30$
$f(2) = -12$
$f(3) = 12$
There is a change of sign (negative to positive) and f is continuous, so there is a solution between x = 2 and x = 3

Answer: Shown: f(2) = −12, f(3) = 12 (change of sign)

8 Reverse compound change

The value of an investment grows by 20% each year. (Calculator)
After 2 years it is worth £48,960.
Work out its value at the start.

Yearly multiplier = 1.2
Overall multiplier = $1.2^2 = 1.44$
Original value = £48,960 ÷ 1.44 = £34,000

Answer: £34,000

9 Rationalising with conjugates

Rationalise the denominator of $\frac{2}{2 - \sqrt{3}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $2 + \sqrt{3}$: $\frac{2(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$

Denominator: 4 − 3 = 1

$\frac{2(2 + \sqrt{3})}{1} = 2(2 + \sqrt{3})$

$= 4 + 2\sqrt{3}$

Answer: $4 + 2\sqrt{3}$

10 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{4x - 1}{-x - 4}$

Multiply both sides by (−x − 4): $y(-x - 4) = 4x - 1$
Expand the bracket: $-xy - 4y = 4x - 1$
Collect the x terms on one side: $-xy - 4x = 4y - 1$
Factorise: $x(-y - 4) = 4y - 1$
Divide by (−y − 4): $x = \frac{4y - 1}{-y - 4}$

Multiply the top and bottom by −1: $x = \frac{-4y + 1}{y + 4}$

Answer: $x = \frac{-4y + 1}{y + 4}$