Day 334 **A Tier 10-a-day: Model Solutions**

1 Algebraic formulae in context

A mechanic charges £35 plus £60 for each hour of work.
(a) Write a formula for the total cost, £C, after t hours.
(b) Chloe pays £575 in total. Work out the number of hours.

(a) C = 35 + 60t
(b) 35 + 60t = 575
60t = 540
t = 9
Answer: (a) C = 35 + 60t (b) 9

2 HCF and LCM

(a) Write 20 and 25 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 20 and 25.

(a) $20 = 2^2 \times 5$ and $25 = 5^2$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: 5
LCM: take the larger power of every prime: $2^2 \times 5^2 = 100$
Answer: (a) $20 = 2^2 \times 5$, $25 = 5^2$ (b) HCF = 5, LCM = 100

3 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked (2x + 25)°.
The diagram is not drawn accurately.

(a) Alternate angles are equal
2x + 25 = 4x − 7
2x = 32
x = 16
(b) 2 × 16 + 25 = 57
Answer: (a) x = 16 (b) 57°

4 Equations with fractions

Solve $\frac{x+8}{3} + \frac{x-1}{6} = 3$

Multiply every term by 6 (the lowest common multiple of 3 and 6)
2(x + 8) + (x − 1) = 6 × 3
3x + 15 = 18
3x = 3
x = 1
Answer: x = 1

5 Capture-recapture

A scientist wants to estimate the number of frogs in a pond.
She catches 20 frogs, marks each one and returns them.
Later she catches a second sample of 100 frogs. 10 of these are marked.
Work out an estimate for the total number of frogs in a pond.

Proportion marked in the second sample = $\frac{10}{100}$

This is the same as the proportion in the whole population: $\frac{20}{N}$

$\frac{20}{N} = \frac{10}{100}$, so N = 20 × 100 ÷ 10 = 200
Answer: 200

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6 Recurring decimals, show that

(a) Show that $0.\dot{1}\dot{0} = \frac{10}{99}$

(b) Hence write $1.\dot{1}\dot{0}$ as a mixed number in its simplest form.

(a) Let x = 0.101010..., so 100x = 10.1010...

100x − x = 10, so 99x = 10 and $x = \frac{10}{99}$

(b) $1.\dot{1}\dot{0} = 1 + \frac{10}{99} = 1\frac{10}{99}$

Answer: (b) $\mathbf{1\frac{10}{99}}$

7 Average speed

A train travels for 30 minutes at 60 km/h,
then for 1 hour 30 minutes at 50 km/h.
Work out the average speed for the whole journey.

Distance 1 = 60 × 0.5 = 30 km
Distance 2 = 50 × 1.5 = 75 km
Total distance = 105 km and total time = 2 hours
Average speed = 105 ÷ 2 = 52.5 km/h
Answer: 52.5 km/h

8 Enlargements and coordinates

Triangle PQR has vertices P(5, 2), Q(−1, 5), R(4, −1).
The triangle is enlarged with scale factor 4 and centre (3, 2).
Write down the coordinates of the vertices of the image.

Each vertex moves to: centre + scale factor × (vertex − centre)
P: (5, 2) → (11, 2)
Q: (−1, 5) → (−13, 14)
R: (4, −1) → (7, −10)
Answer: P'(11, 2), Q'(−13, 14), R'(7, −10)

9 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{5x + 5}{x + 5}$

Multiply both sides by (x + 5): y(x + 5) = 5x + 5
Expand the bracket: xy + 5y = 5x + 5
Collect the x terms on one side: xy − 5x = −5y + 5
Factorise: x(y − 5) = −5y + 5
Divide by (y − 5): $x = \frac{-5y + 5}{y - 5}$

Answer: $\mathbf{x = \frac{-5y + 5}{y - 5}}$

10 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.
Angle BAC = 61° and angle ACD = 48°.
Work out the size of angle x (angle AEB).
Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 48°
In triangle ABE: angle BAE = 61° and angle ABE = 48°
x = 180 − 61 − 48 = 71
Answer: x = 71°