Day 328 **A Tier 10-a-day: Model Solutions**

1 Standard form calculations

Work out $(6 \times 10^{-4}) \div (1.5 \times 10^{-7})$
Give your answer in standard form.

Divide the numbers and subtract the powers of 10: $6 \div 1.5 = 4$ and $10^{-4} \div 10^{-7} = 10^{3}$
4×10^{3}
Answer: 4×10^{3}

2 Converting units of area and volume

Convert 12500 cm^2 into m^2.

$1\ m^2 = 10\ 000\ cm^2$
$12500 \div 10\ 000 = 1.25$
Answer: $1.25\ m^2$

3 Straight line graphs

A straight line has equation $5x - 3y = 20$
(a) Find the gradient of the line.
(b) Find the coordinates of the point where the line crosses the x-axis.

(a) Rearrange into $y = mx + c$: $-3y = -5x + 20$
$y = \frac{5}{3}x - \frac{20}{3}$
The gradient is $\frac{5}{3}$
(b) On the x-axis, $y = 0$: $5x = 20$, so $x = 4$
Answer: (a) $\frac{5}{3}$ (b) (4, 0)

4 Index equations

Solve $3^{x+4} = 9^{x}$

$9 = 3^2$, so $9^x = (3^2)^x = 3^{2x}$
$x + 4 = 2x$
$4 = 2x - x$
$x = 4$
Answer: $x = 4$

5 Venn diagrams from information

There are 29 students in a class.
17 students do tennis and 6 students do badminton.
8 students do neither.
(a) Work out how many students do both.
(b) One student is chosen at random. Find the probability that they do tennis only.

(a) Students doing at least one = $29 - 8 = 21$
Both = $17 + 6 - 21 = 2$
(b) tennis only = $17 - 2 = 15$
Probability = $\frac{15}{29}$
Answer: (a) 2 (b) $\frac{15}{29}$

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6 Expanding three brackets

Expand and simplify
$(x + 3)(x - 4)(x + 5)$

Expand the first two brackets: $(x + 3)(x - 4) = x^2 - x - 12$
Multiply by the third bracket: $(x^2 - x - 12)(x + 5)$
$= x^3 - x^2 - 12x + 5x^2 - 5x - 60$
$= x^3 + 4x^2 - 17x - 60$
Answer: $x^3 + 4x^2 - 17x - 60$

7 Arc length and perimeter of a sector

The diagram shows a sector of a circle of radius 9 cm.
Work out the perimeter of the sector.
Give your answer in terms of π.
The diagram is not drawn accurately.

Arc length $= \frac{80}{360} \times 2 \times \pi \times 9 = 4\pi$
Perimeter = arc + two radii
$= 4\pi + 18$
Answer: $4\pi + 18$ cm

8 Reading a histogram

The histogram shows information about the length of some items.
(a) Work out the number of items with a length in the class 20 to 30 mm.
(b) Work out an estimate for the number with a length greater than 25 mm.

(a) Frequency = frequency density × class width
$0.4 \times 10 = 4$
(b) 25 to 30: $0.4 \times 5 = 2$
Class 30 to 50: $0.8 \times 20 = 16$
Class 50 to 70: $1.4 \times 20 = 28$
Total $= 2 + 16 + 28 = 46$
Answer: (a) 4 (b) 46

9 HCF and LCM in index form

$A = 3 \times 5 \times 7$
$B = 3^3 \times 5^2 \times 7$
Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.
Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): $3 \times 5 \times 7$
LCM: for each prime take the higher power: $3^3 \times 5^2 \times 7$
Answer: HCF = $3 \times 5 \times 7$, LCM = $3^3 \times 5^2 \times 7$

10 Probability with three selections

A bag contains 3 red counters and 5 blue counters.
Three counters are taken at random without replacement.
Work out the probability that at least one of the three counters is blue.

P(none is blue) = P(all red) $= \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{1}{56}$

P(at least one blue) $= 1 - \frac{1}{56} = \frac{55}{56}$

Answer: $\frac{55}{56}$