Day 327 **A Tier 10-a-day: Model Solutions**

1 Functions

f(x) = 4x − 1
g(x) = x^2 + 8
(a) Find f(5)
(b) Find fg(x). Give your answer in its simplest form.

(a) f(5) = 4 × 5 − 1 = 19
(b) fg(x) means f(g(x)), so replace x in f with x^2 + 8
fg(x) = 4(x^2 + 8) − 1
= 4x^2 + 31
Answer: (a) 19 (b) 4x^2 + 31

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.
40% 0.47 $\frac{17}{20}$ $\frac{20}{25}$

Change each to a decimal: 40% = 0.4, 0.47 = 0.47, $\frac{17}{20}$ = 0.85, $\frac{20}{25}$ = 0.8
Order the decimals: 0.4 < 0.47 < 0.8 < 0.85
Answer: 40%, 0.47, $\frac{20}{25}$, $\frac{17}{20}$

3 Capture-recapture

A scientist wants to estimate the number of frogs in a pond.
She catches 20 frogs, marks each one and returns them.
Later she catches a second sample of 40 frogs. 4 of these are marked.
Work out an estimate for the total number of frogs in a pond.

Proportion marked in the second sample = $\frac{4}{40}$
This is the same as the proportion in the whole population: $\frac{20}{N}$
$\frac{20}{N} = \frac{4}{40}$, so N = 20 × 40 ÷ 4 = 200
Answer: 200

4 Congruent triangles

Triangle STU: ST = 11 cm, TU = 14 cm, SU = 10 cm.
Triangle JKL: JK = 14 cm, KL = 10 cm, JL = 11 cm.
Are the two triangles congruent?
Give a reason for your answer.

ST = JL = 11 cm
TU = JK = 14 cm
SU = KL = 10 cm
All three pairs of sides are equal, so the triangles are congruent (SSS)
Answer: Yes, congruent (SSS)

5 Simultaneous equations

Solve the simultaneous equations
−5x − 2y = 14
5x − y = −8

Multiply (2) by 2 to make the y coefficients match
−5x − 2y = 14 (1)
10x − 2y = −16 (2)
The y terms have the same sign, so subtract equation (1) from equation (2)
15x = −30, so x = −2
Substitute x = −2 into equation (1) to find y = −2
Answer: x = −2, y = −2

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6 Compound shapes with circles

The diagram shows a shape made from a rectangle and a semicircle.
Work out the perimeter of the shape.
Give your answer in terms of π.
The diagram is not drawn accurately.

Curved edge: $\frac{1}{2} \times \pi \times 20 = 10\pi$

Straight edges: $20 + 14 + 14 = 48$

Total = $48 + 10\pi$

Answer: $48 + 10\pi$ cm

7 Map scales

The scale of a map is 1 : 50000.
A road is 1.5 km long.
Work out the length of the road on the map, in centimetres.

1.5 km = 150000 cm

Length on map = $150000 \div 50000 = 3$ cm

Answer: 3 cm

8 Area of a sector

The diagram shows a sector of a circle of radius 9 cm.
Work out the area of the sector.
Give your answer in terms of π.
The diagram is not drawn accurately.

Area = $\frac{80}{360} \times \pi \times 9^2$

$= \frac{2}{9} \times 81\ \pi$

$= 18\pi$

Answer: 18π cm^2

9 Addition and multiplication rules

For two events A and B:
P(A) = 0.65, P(B) = 0.65 and P(A or B) = 0.85.
(a) Work out P(A and B).
(b) Work out the probability that neither A nor B happens.

(a) P(A and B) = P(A) + P(B) − P(A or B) = 0.65 + 0.65 − 0.85 = 0.45

(b) 1 − P(A or B) = 1 − 0.85 = 0.15

Answer: (a) 0.45 (b) 0.15

10 Finding a rate of compound change

Mo invests £11,500 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £15,208.75.
Work out the annual rate of interest.

Overall multiplier = £15,208.75 ÷ £11,500 = 1.3225

Take the square root: $\sqrt{1.3225} = 1.15$

Rate = $(1.15 - 1) \times 100 = 15\%$

Answer: 15%