Day 323 **A Tier 10-a-day: Model Solutions**

1 Double inequalities

(a) Solve $-11 < 3x - 5 \leq -2$
(b) Write down all the integer values of x that satisfy this inequality.

(a) Add 5 to every part: $-6 < 3x \leq 3$
Divide every part by 3: $-2 < x \leq 1$
(b) The integers are -1, 0, 1
Answer: (a) $-2 < x \leq 1$ (b) -1, 0, 1

2 Terminating and recurring decimals

Here are four fractions: $\frac{2}{6}$ $\frac{13}{40}$ $\frac{2}{4}$ $\frac{3}{4}$
Which one of these fractions is equal to a recurring decimal?

Simplify each fraction: $\frac{2}{6} = \frac{1}{3}$, $\frac{13}{40}$, $\frac{2}{4} = \frac{1}{2}$, $\frac{3}{4}$
A fraction in simplest form is a terminating decimal only if its denominator has no prime factors other than 2 and 5.
$\frac{1}{3}$ has denominator 3, which includes a factor other than 2 and 5.
Answer: $\frac{2}{6}$

3 Expanding double brackets

Expand and simplify
$(3x - 3)(3x + 4)$

Multiply every term in the first bracket by every term in the second
$= 9x^2 + 12x - 9x - 12$
$= 9x^2 + 3x - 12$
Answer: $9x^2 + 3x - 12$

4 Converting area and volume units

Change 37500 cm^3 to litres.

1 litre = 1000 cm^3
37500 ÷ 1000 = 37.5
Answer: 37.5 litres

5 Reverse percentages

In a sale, all prices are reduced by 6%. (Calculator)
A watch costs £61.10 in the sale.
Work out the original price of the watch.

The sale price is 94% of the original price.
94% = £61.10, so 1% = £61.10 ÷ 94 = £0.65
100% = £0.65 × 100 = £65
Answer: £65

6 Congruent triangles

Triangle STU: angle T = 35°, angle U = 60°, TU = 7 cm.
Triangle DEF: angle E = 60°, angle F = 35°, EF = 7 cm.
Are the two triangles congruent?
Give a reason for your answer.

angle T = angle F = 35°
angle U = angle E = 60°
TU = EF = 7 cm, the side between the two angles
Two angles and the included side are equal, so the triangles are congruent (ASA)
Answer: Yes, congruent (ASA)

Day 323 **A Tier 10-a-day: Model Solutions**

7 Perpendicular bisector

A is the point (3, −5) and B is the point (5, −1).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $(\frac{3+5}{2}, \frac{(-5)+(-1)}{2}) = (4, -3)$

Gradient of AB = $\frac{4}{2} = 2$

Perpendicular gradient = $-1 \div 2 = -\frac{1}{2}$

$y = -\frac{1}{2}x + c$ through (4, −3): $-3 = -\frac{1}{2} \times 4 + c$, so $c = -1$

$y = -\frac{1}{2}x - 1$

Answer: $y = -\frac{1}{2}x - 1$

8 Rates of flow

A tank is in the shape of a cuboid measuring 100 cm by 40 cm by 150 cm.
It already contains 50 litres of water.
Water flows in at 20 litres per minute until the tank is full.
Work out how long this takes. Give your answer in minutes.

Volume = 100 × 40 × 150 = 600000 cm^3 = 600 litres
Water needed = 600 − 50 = 550 litres
Time = 550 ÷ 20 = 27.5 minutes
Answer: 27.5 minutes

9 Bounds and accuracy

A cyclist travels 720 metres in 6.4 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.
(a) Work out the upper bound and the lower bound of the speed in m/s.
(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = 725 ÷ 6.35 = 114.1732
Lower bound = 715 ÷ 6.45 = 110.8527
(b) Both bounds round to 110 to 2 s.f. but not to the same value to 3 s.f.
So the speed is 110 m/s to 2 significant figures.
Answer: (a) 114.1732, 110.8527 (b) 110 m/s (2 s.f.)

10 Volume of cones and spheres

A sphere has radius 5 cm.
Work out the volume of the sphere. Give your answer in terms of π.

Volume = $\frac{4}{3} \times \pi \times r^3$

$= \frac{4}{3} \times \pi \times 5^3$

$= \frac{4}{3} \times 125\,\pi$

$= \frac{500}{3}\pi$

Answer: $\frac{500}{3}\pi$ cm^3