Day 322 **A Tier 10-a-day: Model Solutions**

1 Time

(a) A flight takes off at 6:40 pm. The journey takes 3 hours 44 minutes. Work out the time it ends.
(b) Change 150 minutes into hours. Give your answer as a decimal.

(a) 6:40 pm + 3 hours 44 minutes = 10:24 pm
(b) 150 ÷ 60 = 2.5 hours
Answer: (a) 10:24 pm (b) 2.5 hours

2 Column vectors

$\mathbf{a} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$.
Work out $3\mathbf{a} - 1\mathbf{b}$.

$3\mathbf{a} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$ and $1\mathbf{b} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$3\mathbf{a} - 1\mathbf{b} = \begin{pmatrix} -6 - (-4) \\ 0 - 1 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-2} \\ \mathbf{-1} \end{pmatrix}$

3 Standard form in context

Light travels at 3×10^5 kilometres per second.
The Sun is 9.6×10^7 kilometres from Earth.
Work out how many seconds light takes to travel from the Sun to Earth.
Give your answer in standard form.

time = distance ÷ speed = $(9.6 \times 10^7) \div (3 \times 10^5)$
$9.6 \div 3 = 3.2$ and $10^7 \div 10^5 = 10^2$
$= 3.2 \times 10^2$ seconds
Answer: 3.2×10^2 seconds

4 Area of a sector

The diagram shows a sector of a circle of radius 7 cm.
Work out the area of the sector.
Give your answer in terms of π.
The diagram is not drawn accurately.

Area = $\frac{240}{360} \times \pi \times 7^2$

$= \frac{2}{3} \times 49\,\pi$

$= \frac{98}{3}\pi$

Answer: $\mathbf{\frac{98}{3}\pi}$ **cm^2**

5 Index equations

Solve $3^x = \frac{1}{3}$

Write the right-hand side as a power of 3: $\frac{1}{3} = 3^{-1}$

So $x = -1$
Answer: $\mathbf{x = -1}$

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6 Quadratic sequences

Here are the first five terms of a quadratic sequence.
15, 33, 59, 93, 135
(a) Find an expression for the nth term.
(b) Work out the 10th term.

First differences: 18, 26, 34
Second differences: 8, 8, so a = 8 ÷ 2 = 4
Subtract $4n^2$ from each term (4, 16, 36, 64): 11, 17, 23, 29
This is linear: 6n + 5
nth term = $4n^2 + 6n + 5$
(b) Substitute n = 10: $4 \times 10^2 + 6 \times 10 + 5 = 465$
Answer: (a) $4n^2 + 6n + 5$ (b) 465

7 Inverse proportion formulae

y is inversely proportional to $\sqrt{x}$.
When x = 100, y = 12.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 25.

(a) $y = \frac{k}{\sqrt{x}}$. Substitute: $12 = \frac{k}{10}$, so k = 12 × 10 = 120
$y = \frac{120}{\sqrt{x}}$
(b) $y = \frac{120}{5} = 24$
Answer: (a) $y = \frac{120}{\sqrt{x}}$ (b) 24

8 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{3x + 2}{2x + 6}$

Multiply both sides by (2x + 6): y(2x + 6) = 3x + 2
Expand the bracket: 2xy + 6y = 3x + 2
Collect the x terms on one side: 2xy − 3x = −6y + 2
Factorise: x(2y − 3) = −6y + 2
Divide by (2y − 3): $x = \frac{-6y + 2}{2y - 3}$
Answer: $x = \frac{-6y + 2}{2y - 3}$

9 Changing a ratio

A bag contains red counters and blue counters in the ratio 2 : 5.
15 more red counters are added to the bag.
The ratio of red counters to blue counters is now 9 : 10.
How many counters were in the bag at the start?

Let red = 2x and blue = 5x. After adding: (2x + 15) : 5x = 9 : 10
$\frac{2x + 15}{5x} = \frac{9}{10}$, so 10(2x + 15) = 9 × 5x
20x + 150 = 45x, so 25x = 150, x = 6
Start: 2x + 5x = 7 × 6 = 42 counters
Answer: 42

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10 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.
Angle BAC = 35° and angle ACD = 59°.
Work out the size of angle x (angle AEB).
Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 59°
In triangle ABE: angle BAE = 35° and angle ABE = 59°
x = 180 − 35 − 59 = 86
Answer: x = 86°