Day 32 **A Tier 10-a-day: Model Solutions**

1 Mixed number multiplication and division

Work out $5\frac{3}{5} \times 4\frac{1}{2}$

Give your answer as a mixed number in its simplest form.

Improper fractions: $\frac{28}{5}$ and $\frac{9}{2}$

Multiply: $\frac{28}{5} \times \frac{9}{2} = \frac{252}{10}$

Simplify: $25\frac{1}{5}$

Answer: $25\frac{1}{5}$

2 Map scales

The scale of a map is 1 : 200000.
Two towns are 1.5 cm apart on the map.
Work out the real distance between the towns in kilometres.

Real distance = 1.5 × 200000 = 300000 cm
300000 cm = 3000 m = 3 km
Answer: 3 km

3 Circle theorems: angle at the centre

A, B and C are points on a circle with centre O.
Angle AOB = 60°.
Work out the size of angle x.
Give a reason for your answer.

The angle at the centre is twice the angle at the circumference
x = 60 ÷ 2 = 30
Answer: x = 30°

4 Reverse fractions

$\frac{5}{8}$ of the students in a school are in Year 11.

There are 40 students in Year 11.
How many students are in the school?

$\frac{5}{8}$ of the total = 40

$\frac{1}{8}$ of the total = 40 ÷ 5 = 8

Whole = 8 × 8 = 64
Answer: 64

5 Volume of cones and spheres

A cone has base radius 2 cm and perpendicular height 9 cm.
Work out the volume of the cone. Give your answer in terms of π.

Volume = $\frac{1}{3} \times \pi \times r^2 \times h$

= $\frac{1}{3} \times \pi \times 2^2 \times 9$

= 12π

Answer: 12π cm^3

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6 Simplifying algebraic fractions

Simplify fully

$$\frac{x^2 - 13x + 42}{x^2 - 11x + 28}$$

Factorise the top: $(x - 7)(x - 6)$
Factorise the bottom: $(x - 7)(x - 4)$
Cancel the common factor $(x - 7)$

Answer: $\mathbf{\frac{x - 6}{x - 4}}$

7 Trigonometry in context

A tower stands on horizontal ground. (Calculator)
From a point 90 m from the base of the tower, the angle of elevation of the top is 41°.
Work out the height of the tower. Give your answer to 3 significant figures.

$\tan 41° = \frac{\text{height}}{90}$

height = 90 × tan 41°
= 78.2

Answer: 78.2 m

8 Capture-recapture

A scientist wants to estimate the number of deer in a forest.
She catches 80 deer, marks each one and returns them.
Later she catches a second sample of 30 deer. 10 of these are marked.
Work out an estimate for the total number of deer in a forest.

Proportion marked in the second sample = $\frac{10}{30}$

This is the same as the proportion in the whole population: $\frac{80}{N}$

$\frac{80}{N} = \frac{10}{30}$, so N = 80 × 30 ÷ 10 = 240

Answer: 240

9 Composite functions

f(x) = 3x + 6
$g(x) = x^2 - 5$
Solve gf(x) = 4

$gf(x) = (3x + 6)^2 - 5$
$(3x + 6)^2 - 5 = 4$, so $(3x + 6)^2 = 9$
$3x + 6 = \pm 3$
$3x + 6 = 3$ or $3x + 6 = -3$
$x = -1$ or $x = -3$

Answer: $x = -1$ or $x = -3$

10 Reading a histogram

The histogram shows information about the length of some items.
(a) Work out the number of items with a length in the class 0 to 20 mm.
(b) Work out an estimate for the number with a length greater than 25 mm.

(a) Frequency = frequency density × class width
1.4 × 20 = 28
(b) 25 to 30: 2.2 × 5 = 11
Class 30 to 40: 2.4 × 10 = 24
Class 40 to 60: 1.6 × 20 = 32
Total = 11 + 24 + 32 = 67

Answer: (a) 28 (b) 67