Day 318 **A Tier 10-a-day: Model Solutions**

1 Median from a frequency table

Here are the results for 19 children.
(a) Find the median.
(b) Work out the range.

(a) The median is the 10th value (19 + 1) ÷ 2 = 10
Cumulative frequencies: 2, 5, 11, 18, 19
The 10th value is 2
(b) Range = 4 − 0 = 4
Answer: (a) 2 (b) 4

2 Circle area and circumference

A circular table top has diameter 16 cm.
Work out, in terms of π,
(a) the circumference,
(b) the area.

(a) Circumference = πd = π × 16 = 16π
(b) Area = $\pi r^2 = \pi \times 8^2 = 64\pi$
Answer: (a) 16π cm (b) 64π cm^2

3 Finding a percentage change

The number of members of a club was 240 and is now 96.
Work out the percentage decrease.

Change = 240 − 96 = 144
Percentage decrease = $\frac{144}{240} \times 100 = 60\%$
Answer: 60% decrease

4 Growth and decay

A house is worth £250000. (Calculator)
Its value increases by 10% each year.
Work out its value after 3 years., to the nearest pound.

Multiplier = 100% + 10% = 1.1
$250000 \times 1.1^3 = 332750.00$
= £332750 to the nearest whole number
Answer: £332750

5 Pythagoras in context

A ladder reaches 6 m up a vertical wall.
The ladder is 10 m long.
Work out how far the foot of the ladder is from the wall.

$d^2 + 6^2 = 10^2$
$d^2 = 100 - 36 = 64$
d = 8
Answer: 8 m

6 Quadratic graphs

The curve C has equation $y = -x^2 + 2x + 8$
(a) Find the coordinates of the points where C crosses the x-axis.
(b) Find the coordinates of the turning point of C.

(a) Set y = 0: $-x^2 + 2x + 8 = 0$, so $x^2 - 2x - 8 = 0$ (divide by −1)
(x + 2)(x − 4) = 0
x = −2 or x = 4, so the graph crosses the x-axis at (−2, 0) and (4, 0)
(b) The turning point is halfway between the roots: x = (−2 + 4) ÷ 2 = 1
y = 9, so the maximum point is (1, 9)
Answer: (a) (−2, 0) and (4, 0) (b) (1, 9)

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7 Rates of flow

A tank is in the shape of a cuboid measuring 100 cm by 60 cm by 40 cm.
It already contains 100 litres of water.
Water flows in at 10 litres per minute until the tank is full.
Work out how long this takes. Give your answer in minutes.

Volume = $100 \times 60 \times 40 = 240000$ cm^3 = 240 litres
Water needed = $240 - 100 = 140$ litres
Time = $140 \div 10 = 14$ minutes
Answer: 14 minutes

8 Percentages of percentages

There are 200 passengers on a plane.
60% of the passengers are children.
30% of the children are under 5 years old.
How many passengers are children under 5?

Children = 60% of 200 = 120
Under 5 = 30% of 120 = 36
Answer: 36

9 Circles and tangents

The point P(−12, −5) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = (-12)^2 + (-5)^2 = 169$

(b) Gradient of the radius OP = $\frac{-5}{-12} = \frac{5}{12}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div \frac{5}{12} = -\frac{12}{5}$

$y = -\frac{12}{5}x + c$ through (−12, −5): $-5 = -\frac{12}{5} \times (-12) + c$, so $c = -\frac{169}{5}$

$y = -\frac{12}{5}x - \frac{169}{5}$

Answer: (a) $x^2 + y^2 = 169$ (b) $y = -\frac{12}{5}x - \frac{169}{5}$

10 Bounds and accuracy

A cyclist travels 190 metres in 17.4 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.
(a) Work out the upper bound and the lower bound of the speed in m/s.
(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = $195 \div 17.35 = 11.2392$
Lower bound = $185 \div 17.45 = 10.6017$
(b) Both bounds round to 11 to 2 s.f. but not to the same value to 3 s.f.
So the speed is 11 m/s to 2 significant figures.
Answer: (a) 11.2392, 10.6017 (b) 11 m/s (2 s.f.)