Day 313 **A Tier 10-a-day: Model Solutions**

1 Circle area and circumference

A semicircle has diameter 8 cm.
Work out, in terms of π,
(a) the perimeter of the semicircle,
(b) the area of the semicircle.

(a) Curved edge = $\frac{1}{2} \times \pi \times 8 = 4\pi$

Perimeter = $4\pi + 8$

(b) Area = $\frac{1}{2} \times \pi \times 4^2 = 8\pi$

Answer: (a) $4\pi + 8$ cm (b) 8π cm^2

2 Time

(a) A film starts at 1155. The film lasts 3 hours 56 minutes. Work out the time it ends.
(b) Change 3.3 hours into hours and minutes.

(a) 1155 + 3 hours = 1455, then + 56 minutes = 1551
(b) 0.3 hours = 0.3 × 60 = 18 minutes, so 3.3 hours = 3 hours 18 minutes
Answer: (a) 1551 (b) 3 hours 18 minutes

3 Double inequalities

(a) Solve $-1 < 4x - 5 \leq 27$
(b) Write down all the integer values of x that satisfy this inequality.

(a) Add 5 to every part: $4 < 4x \leq 32$
Divide every part by 4: $1 < x \leq 8$
(b) The integers are 2, 3, 4, 5, 6, 7, 8
Answer: (a) $1 < x \leq 8$ (b) 2, 3, 4, 5, 6, 7, 8

4 Negative and fractional indices

Work out the value of

(a) $4^{\frac{3}{2}}$

(b) $216^{\frac{-1}{3}}$

(a) $4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$

(b) $216^{\frac{-1}{3}} = \frac{1}{216^{\frac{1}{3}}} = \frac{1}{(\sqrt[3]{216})} = \frac{1}{6}$

Answer: (a) 8 (b) $\frac{1}{6}$

5 Finding a missing value from an average

The mean of the five numbers 25, 12, x, 21, 4 is 14.
Work out the value of x.

Total = 5 × 14 = 70
Known numbers add to 62
x = 70 − 62 = 8
Answer: x = 8

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6 Parallel and perpendicular lines

The line L has equation $y = \frac{3}{2}x - 4$

Find the equation of the line that is perpendicular to L and passes through the point (0, 4).

The gradient of L is $\frac{3}{2}$

Perpendicular gradient = $-1 \div \frac{3}{2} = -\frac{2}{3}$

$y = -\frac{2}{3}x + c$, so $4 = -\frac{2}{3} \times 0 + c$

$c = 4$

$y = -\frac{2}{3}x + 4$

Answer: $y = -\frac{2}{3}x + 4$

7 Reading a histogram

The histogram shows information about the mass of some items.
(a) Work out the number of items with a mass in the class 40 to 50 g.
(b) Work out an estimate for the number with a mass greater than 45 g.

(a) Frequency = frequency density × class width
1.8 × 10 = 18
(b) 45 to 50: 1.8 × 5 = 9
Class 50 to 60: 0.6 × 10 = 6
Total = 9 + 6 = 15
Answer: (a) 18 (b) 15

8 Surface area of cones and spheres

A solid cone has base radius 8 cm and perpendicular height 6 cm.
Work out the total surface area of the cone.
Give your answer in terms of π.

Slant height $l = \sqrt{8^2 + 6^2} = 10$
Curved surface = $\pi rl = 80\pi$
Base = $\pi r^2 = 64\pi$
Total = $80\pi + 64\pi = 144\pi$
Answer: 144π cm^2

9 Frequency density

The table gives information about 100 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $20 < m \leq 35$.

(a) Missing frequency = 100 − (16 + 42 + 12) = 30
(b) Class width = 35 − 20 = 15
Frequency density = 42 ÷ 15 = 2.8
Answer: (a) 30 (b) 2.8

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10 Bounds and accuracy

A cyclist travels 380 metres in 16.5 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.

(a) Work out the upper bound and the lower bound of the speed in m/s.

(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = 385 ÷ 16.45 = 23.4043

Lower bound = 375 ÷ 16.55 = 22.6586

(b) Both bounds round to 23 to 2 s.f. but not to the same value to 3 s.f.

So the speed is 23 m/s to 2 significant figures.

Answer: (a) 23.4043, 22.6586 (b) 23 m/s (2 s.f.)