Day 310 **A Tier 10-a-day: Model Solutions**

1 Volume of a prism

The diagram shows a prism.
The cross-section of the prism is a trapezium.
Work out the volume of the prism.
The diagram is not drawn accurately.

Area of cross-section = $\frac{1}{2} \times (6 + 14) \times 9 = 90$

Volume = area of cross-section × length
= 90 × 19
= 1710

Answer: 1710 cm^3

2 HCF and LCM

(a) Write 27 and 30 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 27 and 30.

(a) $27 = 3^3$ and $30 = 2 \times 3 \times 5$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: 3
LCM: take the larger power of every prime: $2 \times 3^3 \times 5 = 270$

Answer: (a) $27 = 3^3$, $30 = 2 \times 3 \times 5$ (b) HCF = 3, LCM = 270

3 Inequalities

Solve $2x + 9 < -x + 6$

Collect the x terms on the left: $3x + 9 < 6$
Subtract 9: $3x < -3$
Divide by 3
$x < -1$

Answer: $x < -1$

4 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked $(4x - 5)°$.
The diagram is not drawn accurately.

(a) Alternate angles are equal
$4x - 5 = 2x + 23$
$2x = 28$
$x = 14$
(b) $4 \times 14 - 5 = 51$

Answer: (a) x = 14 (b) 51°

5 Rates of flow

A tank is in the shape of a cuboid measuring 40 cm by 100 cm by 120 cm.
It already contains 20 litres of water.
Water flows in at 8 litres per minute until the tank is full.
Work out how long this takes. Give your answer in minutes.

Volume = $40 \times 100 \times 120 = 480000$ cm^3 = 480 litres
Water needed = 480 − 20 = 460 litres
Time = 460 ÷ 8 = 57.5 minutes

Answer: 57.5 minutes

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6 Surd fractions

Show that $\frac{\sqrt{125}-\sqrt{45}}{\sqrt{5}}$ is an integer, and find its value.

$\sqrt{125} = 5\sqrt{5}$ and $\sqrt{45} = 3\sqrt{5}$

Numerator $= 5\sqrt{5} - 3\sqrt{5} = 2\sqrt{5}$

$\frac{2\sqrt{5}}{\sqrt{5}} = 2$

Answer: 2

7 Expanding three brackets

Expand and simplify

$(x + 1)(x - 3)(x + 3)$

Expand the first two brackets: $(x + 1)(x - 3) = x^2 - 2x - 3$

Multiply by the third bracket: $(x^2 - 2x - 3)(x + 3)$

$= x^3 - 2x^2 - 3x + 3x^2 - 6x - 9$

$= x^3 + x^2 - 9x - 9$

Answer: $x^3 + x^2 - 9x - 9$

8 Finding an interest rate

Ravi invests £2000 in a savings account. (Calculator)
The account pays compound interest at a fixed rate each year.
After 2 years there is £2247.20 in the account.
Work out the annual rate of interest.

Let the yearly multiplier be m. Then $2000 \times m^2 = 2247.2$

$m^2 = 2247.2 \div 2000 = 1.1236$

$m = \sqrt{1.1236} = 1.06$

Rate $= (1.06 - 1) \times 100 = 6\%$

Answer: 6%

9 Area of a segment

The diagram shows a circle with centre O and radius 6 cm.
Angle AOB = 60°. (Calculator)
Work out the area of the shaded segment.
Give your answer to 3 significant figures.

Area of sector $= \frac{60}{360} \times \pi \times 6^2 = 18.850$

Area of triangle OAB $= \frac{1}{2} \times 6 \times 6 \times \sin 60° = 15.588$

Segment $= 18.850 - 15.588 = 3.26$

Answer: 3.26 cm^2

10 Distance-time graphs

A cyclist makes a journey. The distance-time graph shows the journey.
(a) For how many minutes did the cyclist stop?
(b) Work out the speed during the first part of the journey. Give your answer in km/h.

(a) The graph is horizontal from 60 to 70 minutes, so the stop is $70 - 60 = 10$ minutes

(b) First part: distance 15 km in 60 minutes

Speed = distance ÷ time $= 15 \div \frac{60}{60} = 15$ km/h

Answer: (a) 10 minutes (b) 15 km/h