Day 309 **A Tier 10-a-day: Model Solutions**

1 Venn diagrams from information

There are 29 students in a class.
20 students do drama and 11 students do choir.
3 students do neither.
(a) Work out how many students do both.
(b) One student is chosen at random. Find the probability that they do drama only.

(a) Students doing at least one = 29 − 3 = 26
Both = 20 + 11 − 26 = 5
(b) drama only = 20 − 5 = 15
Probability = $\frac{15}{29}$

Answer: (a) 5 (b) $\frac{15}{29}$

2 Expanding double brackets

Expand and simplify
$(x - 3)(2x - 3)$

Multiply every term in the first bracket by every term in the second
$= 2x^2 - 3x - 6x + 9$
$= 2x^2 - 9x + 9$
Answer: $2x^2 - 9x + 9$

3 Pythagoras' theorem

Work out the length of x. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$10^2 = x^2 + 7^2$
$x^2 = 100 - 49 = 51$
$x = \sqrt{51} = 7.14$
Answer: x = 7.14 m

4 Powers and roots in standard form

Work out $(2 \times 10^2)^2$
Give your answer in standard form.

$(2 \times 10^2)^2 = 2^2 \times (10^2)^2$
$= 4 \times 10^4$
Answer: 4×10^4

5 Reverse percentages

The price of a train ticket increases by 20%.
The new price is £480.
Work out the price before the increase.

120% represents £480
1% = £480 ÷ 120 = £4
100% = £4 × 100 = £400.00
Answer: £400.00

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6 Similar shapes: area and volume

Two similar model boats have volumes 8 cm^3 and 64 cm^3.
The smaller one has height 5 cm.
Work out the height of the larger one.

Volume scale factor = 64 ÷ 8 = 8
Length scale factor = cube root of 8 = 2
Height = 5 × 2 = 10
Answer: 10 cm

7 Surd fractions

Show that $\frac{\sqrt{50}+\sqrt{72}}{\sqrt{2}}$ is an integer, and find its value.

$\sqrt{50} = 5\sqrt{2}$ and $\sqrt{72} = 6\sqrt{2}$
Numerator = $5\sqrt{2} + 6\sqrt{2} = 11\sqrt{2}$
$\frac{11\sqrt{2}}{\sqrt{2}} = 11$
Answer: 11

8 Circles and tangents

The point P(4, −3) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 4^2 + (-3)^2 = 25$

(b) Gradient of the radius OP = $\frac{-3}{4}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div (-\frac{3}{4}) = \frac{4}{3}$

$y = \frac{4}{3}x + c$ through (4, −3): $-3 = \frac{4}{3} \times 4 + c$, so $c = -\frac{25}{3}$

$y = \frac{4}{3}x - \frac{25}{3}$

Answer: (a) $x^2 + y^2 = 25$ (b) $y = \frac{4}{3}x - \frac{25}{3}$

9 HCF and LCM

(a) Write 132 and 48 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 132 and 48.

(a) $132 = 2^2 \times 3 \times 11$ and $48 = 2^4 \times 3$

(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2^2 \times 3 = 12$

LCM: take the larger power of every prime: $2^4 \times 3 \times 11 = 528$

Answer: (a) $132 = 2^2 \times 3 \times 11$, $48 = 2^4 \times 3$ (b) HCF = 12, LCM = 528

10 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.
Angle BAC = 43° and angle ACD = 41°.
Work out the size of angle x (angle AEB).
Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 41°
In triangle ABE: angle BAE = 43° and angle ABE = 41°
x = 180 − 43 − 41 = 96
Answer: x = 96°