Day 306 **A Tier 10-a-day: Model Solutions**

1 Standard form calculations

Work out $(4.5 \times 10^{-4}) \div (3 \times 10^{3})$
Give your answer in standard form.

Divide the numbers and subtract the powers of 10: $4.5 \div 3 = 1.5$ and $10^{-4} \div 10^{3} = 10^{-7}$
1.5×10^{-7}
Answer: 1.5×10^{-7}

2 Expanding double brackets

Expand and simplify
$(2x + 2)^2$

Multiply every term in the first bracket by every term in the second
$= (2x + 2)(2x + 2) = 4x^2 + 4x + 4x + 4$
$= 4x^2 + 8x + 4$
Answer: $4x^2 + 8x + 4$

3 Decimal calculations

(a) Work out 0.7×0.08
(b) Work out $0.18 \div 0.03$

(a) $7 \times 8 = 56$, and there are 3 decimal places in total, so 0.056
(b) Multiply both numbers by 100: $18 \div 3 = 6$
Answer: (a) 0.056 (b) 6

4 Rates of flow

A tank is in the shape of a cuboid measuring 60 cm by 50 cm by 40 cm.
It already contains 20 litres of water.
Water flows in at 20 litres per minute until the tank is full.
Work out how long this takes. Give your answer in minutes.

Volume $= 60 \times 50 \times 40 = 120000$ cm^3 = 120 litres
Water needed $= 120 - 20 = 100$ litres
Time $= 100 \div 20 = 5$ minutes
Answer: 5 minutes

5 Volume of a cylinder

A tin of soup is a cylinder with radius 2 cm and height 5 cm.
Work out the volume. Give your answer in terms of π.

Volume $= \pi r^2 h$
$= \pi \times 2^2 \times 5$
$= 20\pi$
Answer: 20π cm^3

6 Inverse proportion formulae

y is inversely proportional to x.
When x = 20, y = 10.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 2.

(a) $y = \frac{k}{x}$. Substitute: $10 = \frac{k}{20}$, so $k = 10 \times 20 = 200$
$y = \frac{200}{x}$
(b) $y = \frac{200}{2} = 100$
Answer: (a) $y = \frac{200}{x}$ (b) 100

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7 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + 3x - 7 = 0$ can be rearranged to give $x = \sqrt[3]{7 - 3x}$
(b) Show that the equation has a solution between x = 1 and x = 2

(a) Add 7 and subtract 3x from both sides: $x^3 = 7 - 3x$
Take the cube root of both sides: $x = \sqrt[3]{7 - 3x}$
(b) Let $f(x) = x^3 + 3x - 7$
$f(1) = -3$
$f(2) = 7$
There is a change of sign (negative to positive) and f is continuous, so there is a solution between x = 1 and x = 2
Answer: Shown: f(1) = −3, f(2) = 7 (change of sign)

8 Interior angles of polygons

The size of each angle of a quadrilateral is shown as an expression:
(3x − 25)°, (x + 20)°, (x + 55)°, (x + 64)°
Work out the value of x.

Sum of angles = (4 − 2) × 180 = 360°
3x − 25 + x + 20 + x + 55 + x + 64 = 360
6x + 114 = 360
6x = 246
x = 41
Answer: x = 41

9 Reverse compound change

The value of a house increases by 5% each year. (Calculator)
After 2 years it is worth £2,205.
Work out its value at the start.

Yearly multiplier = 1.05
Overall multiplier = $1.05^2 = 1.1025$
Original value = £2,205 ÷ 1.1025 = £2,000
Answer: £2,000

10 Finding an interest rate

Zac invests £1000 in a savings account. (Calculator)
The account pays compound interest at a fixed rate each year.
After 2 years there is £1040.40 in the account.
Work out the annual rate of interest.

Let the yearly multiplier be m. Then $1000 \times m^2 = 1040.4$
$m^2 = 1040.4 \div 1000 = 1.0404$
$m = \sqrt{1.0404} = 1.02$
Rate = (1.02 − 1) × 100 = 2%
Answer: 2%