Day 303 **A Tier 10-a-day: Model Solutions**

1 Volume of a pyramid

The diagram shows a pyramid with a square base of side 3 cm.
The perpendicular height of the pyramid is 9 cm.
Work out the volume of the pyramid.
The diagram is not drawn accurately.

Volume = $\frac{1}{3}$ × base area × height

= $\frac{1}{3}$ × 3 × 3 × 9

= 27

Answer: 27 cm^3

2 Double inequalities

(a) Solve $10 \leq 5x + 5 < 25$
(b) Write down all the integer values of x that satisfy this inequality.

(a) Subtract 5 from every part: $5 \leq 5x < 20$
Divide every part by 5: $1 \leq x < 4$
(b) The integers are 1, 2, 3
Answer: (a) $1 \leq x < 4$ (b) 1, 2, 3

3 Median from a frequency table

The frequency table shows data for 23 children.
(a) Find the median.
(b) Work out the range.

(a) The median is the 12th value (23 + 1) ÷ 2 = 12
Cumulative frequencies: 1, 5, 10, 16, 23
The 12th value is 3
(b) Range = 4 − 0 = 4
Answer: (a) 3 (b) 4

4 Compound interest

Mo invests £1,500 in a savings account. (Calculator)
The account pays compound interest at 3.5% per year.
Work out the total interest earned after 4 years.

Multiplier = 1 + 3.5 ÷ 100 = 1.035
£1,500 × 1.035^4 = £1,721.28
Interest = £1,721.28 − £1,500 = £221.28
Answer: £221.28

5 Capture-recapture

A scientist wants to estimate the number of deer in a forest.
She catches 40 deer, marks each one and returns them.
Later she catches a second sample of 50 deer. 10 of these are marked.
Work out an estimate for the total number of deer in a forest.

Proportion marked in the second sample = $\frac{10}{50}$

This is the same as the proportion in the whole population: $\frac{40}{N}$

$\frac{40}{N} = \frac{10}{50}$, so N = 40 × 50 ÷ 10 = 200
Answer: 200

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6 Percentages of percentages

In a school, 40% of the students are boys. (Calculator)
30% of the boys and 30% of the girls walk to school.
What percentage of all the students walk to school?

Take 400 students: boys = 160, girls = 240
Boys who walk = 160 × 0.3 = 48
Girls who walk = 240 × 0.3 = 72
Total = 120, so 120 ÷ 400 × 100 = 30%
Answer: 30%

7 Simplifying algebraic fractions

Simplify fully

$$\frac{x^2 - 6x + 8}{x^2 - 4}$$

Factorise the top: $(x - 2)(x - 4)$
Factorise the bottom: $(x - 2)(x + 2)$
Cancel the common factor $(x - 2)$
Answer: $\mathbf{\frac{x - 4}{x + 2}}$

8 Bearings and distances

A plane leaves an airport and travels 18 km on a bearing of 140°. (Calculator)
It then travels 24 km on a bearing of 050°.
(a) Work out how far it is now from the airport.
(b) Work out the bearing of its final position from the airport. Give your answer to the nearest degree.

The two bearings differ by 90°, so the path makes a right angle
(a) Distance^2 = $18^2 + 24^2 = 900$, so distance = 30 km
(b) The angle at the port between the first leg and the line to the end: $\tan^{-1}(\frac{24}{18}) = 53.1°$
Bearing = 140° − 53.1° = 86.9°
Bearing = 087°
Answer: (a) 30 km (b) 087°

9 Volume of composite solids

A solid is made from a cylinder and a hemisphere.
The cylinder has radius 9 cm and height 6 cm.
The hemisphere has the same radius as the cylinder and sits on top of it.
Work out the total volume. Give your answer in terms of π.

Cylinder = $\pi \times 9^2 \times 6 = 486\pi$
Hemisphere = $\frac{2}{3} \times \pi \times 9^3 = 486\pi$
Total = $486\pi + 486\pi = 972\pi$
Answer: $\mathbf{972\pi}$ cm^3

10 Recurring decimals (mixed)

Write $0.7\dot{6}$ as a fraction in its simplest form.

Let x = 0.766...
Multiply by 10: 10x = 7.66...
Multiply by 100: 100x = 76.66...
Subtract: 90x = 76 − 7 = 69
$x = \frac{69}{90} = \frac{23}{30}$
Answer: $\mathbf{\frac{23}{30}}$