Day 30 **A Tier 10-a-day: Model Solutions**

1 HCF and LCM

(a) Write 120 and 108 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 120 and 108.

(a) $120 = 2^3 \times 3 \times 5$ and $108 = 2^2 \times 3^3$

(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2^2 \times 3 = 12$

LCM: take the larger power of every prime: $2^3 \times 3^3 \times 5 = 1080$

Answer: (a) $120 = 2^3 \times 3 \times 5$, $108 = 2^2 \times 3^3$ (b) HCF = 12, LCM = 1080

2 Straight line graphs

A straight line has equation $5x - 2y = 10$
(a) Find the gradient of the line.
(b) Find the coordinates of the point where the line crosses the x-axis.

(a) Rearrange into $y = mx + c$: $-2y = -5x + 10$

$y = \frac{5}{2}x - 5$

The gradient is $\frac{5}{2}$

(b) On the x-axis, $y = 0$: $5x = 10$, so $x = 2$

Answer: (a) $\frac{5}{2}$ (b) (2, 0)

3 Reverse percentages

Hugo's weekly pay is increased by 12%.
Her new weekly pay is £28.
Work out her weekly pay before the increase.

112% represents £28
1% = £28 ÷ 112 = £0.25
100% = £0.25 × 100 = £25.00
Answer: £25.00

4 Finding a percentage change

The number of visitors to a museum was 400 and is now 500.
Work out the percentage increase.

Change = 500 − 400 = 100

Percentage increase = $\frac{100}{400} \times 100 = 25\%$

Answer: 25% increase

5 Distance-time graphs

A cyclist makes a journey. The distance-time graph shows the journey.
(a) For how many minutes did the cyclist stop?
(b) Work out the speed during the first part of the journey. Give your answer in km/h.

(a) The graph is horizontal from 60 to 70 minutes, so the stop is 70 − 60 = 10 minutes
(b) First part: distance 12 km in 60 minutes

Speed = distance ÷ time = $12 \div \frac{60}{60} = 12$ km/h

Answer: (a) 10 minutes (b) 12 km/h

6 Error intervals

L = 3.15 has been truncated to 2 decimal places.
Write down the error interval for L.

Truncating means cutting off the digits after the second decimal place.
The original value is at least 3.15 and less than 3.15 + 0.01 = 3.16
Answer: $3.15 \leq L < 3.16$

7 Volume of cones and spheres

A solid hemisphere has radius 3 cm.
Work out its volume. Give your answer in terms of π.

A hemisphere is half a sphere

Volume = $\frac{1}{2} \times \frac{4}{3} \times \pi \times 3^3$

= $\frac{2}{3} \times 27\ \pi$

= 18π

Answer: 18π cm^3

8 Equations with algebraic fractions

Solve $\frac{3}{(x+3)} + \frac{3}{(x-5)} = 1$

Multiply every term by $(x + 3)(x - 5)$
$3(x - 5) + 3(x + 3) = (x + 3)(x - 5)$
$6x - 6 = x^2 - 2x - 15$
$x^2 - 8x - 9 = 0$
$(x + 1)(x - 9) = 0$
$x = -1$ or $x = 9$
Answer: $x = -1$ or $x = 9$

9 Volume of a prism

The diagram shows a prism with a right-angled triangle as its cross-section.
The volume of the prism is 605 cm^3.
Work out the value of x.
The diagram is not drawn accurately.

Area of cross-section = $\frac{1}{2} \times 11 \times 10 = 55$

Volume = area × length
605 = 55 × length
length = 605 ÷ 55 = 11
Answer: x = 11

10 Forming quadratic equations

A rectangle has width x cm and length (x + 1) cm.
The area of the rectangle is 72 cm^2.
(a) Show that $x^2 + 1x - 72 = 0$
(b) Work out the value of x.

(a) Area = length × width, so $x(x + 1) = 72$
$x^2 + 1x = 72$
$x^2 + 1x - 72 = 0$
(b) Factorise: $(x - 8)(x + 9) = 0$
$x = 8$ or $x = -9$
A length cannot be negative, so $x = 8$
Answer: (a) Shown (b) x = 8