Day 298 **A Tier 10-a-day: Model Solutions**

1 Direct and inverse proportion

y is inversely proportional to the square of x.
When x = 3, y = 8.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 2.

(a) $y = \frac{k}{x^2}$. Substitute x = 3 and y = 8: $8 = \frac{k}{9}$, so $k = 8 \times 9 = 72$

$y = \frac{72}{x^2}$

(b) $y = \frac{72}{4} = 18$

Answer: (a) $y = \frac{72}{x^2}$ (b) 18

2 Speed, distance and time

A cyclist rides 33 km at 12 km/h.
Work out the time taken. Give your answer in hours and minutes.

Time = distance ÷ speed = 33 ÷ 12 = 2.75 hours
2.75 hours = 2 hours 45 minutes
Answer: 2 hours 45 minutes

3 Rearranging formulae

Make r the subject of $k = \frac{r + 1}{2}$

Multiply both sides by 2: 2k = r + 1
Subtract 1 from both sides
r = 2k − 1
Answer: r = 2k − 1

4 Profit and loss

Ryan buys a bike for £200 and sells it making a profit of 8%.
Work out the selling price.

Selling price = 108% of the cost price
£200 × 1.08 = £216
Answer: £216

5 Pythagoras in context

A ladder reaches 9 m up a vertical wall.
The ladder is 41 m long.
Work out how far the foot of the ladder is from the wall.

$d^2 + 9^2 = 41^2$
$d^2 = 1681 - 81 = 1600$
d = 40
Answer: 40 m

6 Compound interest

Jack invests £3,000 in a savings account. (Calculator)
The account pays compound interest at 2% per year.
Work out the value of the investment after 5 years.

Multiplier = 1 + 2 ÷ 100 = 1.02
$£3{,}000 \times 1.02^5 = £3{,}312.24$
Answer: £3,312.24

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7 Mixing in a ratio

Jug A contains 14 litres of a mixture of red paint and white paint in the ratio 3 : 4.
Jug B contains 36 litres of a mixture of red paint and white paint in the ratio 5 : 1.
The contents of both jugs are mixed together.
Work out the ratio of red paint to white paint in the mixture. Give your answer in its simplest form.

Jug A: red paint = 3 × 14 ÷ 7 = 6 litres, white paint = 8 litres
Jug B: red paint = 30 litres, white paint = 6 litres
Total: red paint = 36 litres, white paint = 14 litres
Ratio = 36 : 14 = 18 : 7
Answer: 18 : 7

8 Surface area of cones and spheres

A sphere has radius 8 cm.
Work out the surface area of the sphere.
Give your answer in terms of π.

Surface area = $4\pi r^2$
= $4 \times \pi \times 8^2$
= 256π
Answer: 256π cm^2

9 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{3x - 3}{-x + 5}$

Multiply both sides by $(-x + 5)$: $y(-x + 5) = 3x - 3$
Expand the bracket: $-xy + 5y = 3x - 3$
Collect the x terms on one side: $-xy - 3x = -5y - 3$
Factorise: $x(-y - 3) = -5y - 3$
Divide by $(-y - 3)$: $x = \frac{-5y - 3}{-y - 3}$
Multiply the top and bottom by -1: $x = \frac{5y + 3}{y + 3}$
Answer: $x = \frac{5y + 3}{y + 3}$

10 Rationalising with conjugates

Rationalise the denominator of $\frac{1}{2 - \sqrt{3}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $2 + \sqrt{3}$: $\frac{1(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$
Denominator: $4 - 3 = 1$
$\frac{1(2 + \sqrt{3})}{1} = 1(2 + \sqrt{3})$
$= 2 + \sqrt{3}$
Answer: $2 + \sqrt{3}$