Day 296 **A Tier 10-a-day: Model Solutions**

1 Quadratic graphs

The curve C has equation $y = x^2 - 6x + 5$
(a) Find the coordinates of the points where C crosses the x-axis.
(b) Find the coordinates of the turning point of C.

(a) Set $y = 0$: $x^2 - 6x + 5 = 0$
$(x - 5)(x - 1) = 0$
$x = 1$ or $x = 5$, so the graph crosses the x-axis at (1, 0) and (5, 0)
(b) The turning point is halfway between the roots: $x = (1 + 5) \div 2 = 3$
$y = -4$, so the minimum point is $(3, -4)$
Answer: (a) (1, 0) and (5, 0) (b) (3, −4)

2 LCM in context

Aisha runs around a track in 90 seconds.
Mo runs around the same track in 75 seconds.
They start together at the start line and keep running.
How many laps will each have run when they are next together at the start line?

Find the LCM of 90 and 75: LCM = 450 seconds.
Aisha: $450 \div 90 = 5$ laps
Mo: $450 \div 75 = 6$ laps
Answer: Aisha 5 laps, Mo 6 laps

3 nth term of a linear sequence

Here are the first four terms of a sequence.
6, 8, 10, 12
(a) Find an expression for the nth term.
(b) Work out the 50th term.

The terms go up by 2 each time, so the nth term starts with 2n
$6 - 2 = 4$, so the nth term is $2n + 4$
(b) $50 \times 2 + 4 = 104$
Answer: (a) 2n + 4 (b) 104

4 Negative and fractional indices

Work out the value of

(a) $(\frac{3}{4})^{-2}$

(b) $16^{\frac{-1}{4}}$

(a) $(\frac{3}{4})^{-2} = (\frac{4}{3})^{2} = \frac{4^2}{3^2} = \frac{16}{9}$

(b) $16^{\frac{-1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{(\sqrt[4]{16})} = \frac{1}{2}$

Answer: (a) $\frac{16}{9}$ (b) $\frac{1}{2}$

5 Compound shapes with circles

The diagram shows a shape made from a rectangle and a semicircle.
Work out the perimeter of the shape.
Give your answer in terms of π.
The diagram is not drawn accurately.

Curved edge: $\frac{1}{2} \times \pi \times 18 = 9\pi$

Straight edges: 18 + 9 + 9 = 36
Total = $36 + 9\pi$
Answer: $36 + 9\pi$ cm

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6 Venn diagrams from information

There are 37 students in a class.
23 students do drama and 19 students do choir.
4 students do neither.
(a) Work out how many students do both.
(b) One student is chosen at random. Find the probability that they do drama only.

(a) Students doing at least one = 37 − 4 = 33
Both = 23 + 19 − 33 = 9
(b) drama only = 23 − 9 = 14
Probability = $\frac{14}{37}$

Answer: (a) 9 (b) $\frac{14}{37}$

7 Expanding three brackets

Expand and simplify
(x − 1)(2x + 1)(x − 1)

Expand the first two brackets: $(x - 1)(2x + 1) = 2x^2 - x - 1$
Multiply by the third bracket: $(2x^2 - x - 1)(x - 1)$
$= 2x^3 - x^2 - x - 2x^2 + x + 1$
$= 2x^3 - 3x^2 + 1$
Answer: $2x^3 - 3x^2 + 1$

8 Best buys

A shop sells rice in 3 different sizes.
Small bag: 200 g for £1.70
Medium bag: 250 g for £1.85
Large bag: 500 g for £3.85
Which bag is the best value for money? You must show your working.

Small bag: 170p ÷ 2 = 85p per 100 g
Medium bag: 185p ÷ 2.5 = 74p per 100 g
Large bag: 385p ÷ 5 = 77p per 100 g
The medium bag has the lowest price per 100 g, so it is the best value
Answer: The medium bag

9 Vector geometry

OAB is a triangle. →OA = **a** and →OB = **b**.
P is the point on AB such that AP : PB = 1 : 4.
Find →OP in terms of **a** and **b**.

$\rightarrow AB = -\mathbf{a} + \mathbf{b}$
$\rightarrow AP = \frac{1}{5}\rightarrow AB = \frac{1}{5}(\mathbf{b} - \mathbf{a})$
$\rightarrow OP = \rightarrow OA + \rightarrow AP = \mathbf{a} + \frac{1}{5}(\mathbf{b} - \mathbf{a})$
$= \frac{4}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$

Answer: $\rightarrow OP = \frac{4}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$

10 Calculator and significant figures

Work out $2.5 \times (1 + 2.7 \div 100)^3$ (Calculator)
Give your answer correct to 3 significant figures.

1 + 2.7 ÷ 100 = 1.027
= 2.70802...
= 2.71 to 3 significant figures
Answer: 2.71