Day 295 **A Tier 10-a-day: Model Solutions**

1 Primes

(a) Write down all the prime numbers between 20 and 40.
(b) Write 95 as the product of two prime numbers.

(a) A prime number has exactly two factors. Checking each number between 20 and 40 gives: 23, 29, 31, 37
(b) 95 ÷ 5 = 19, and both 5 and 19 are prime.
Answer: (a) 23, 29, 31, 37 (b) 5 × 19

2 Circle theorems: angle in a semicircle

AB is a diameter of the circle.
Angle BAC = 31°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle ACB = 90°
Angles in a triangle add up to 180°
x = 180 − 90 − 31 = 59
Answer: x = 59°

3 Percentage change

The population of a village changes from 400 to 384.
Work out the percentage decrease.

Decrease = 400 − 384 = 16

Percentage decrease = $\frac{16}{400} \times 100 = 4\%$

Answer: 4%

4 Expanding three brackets

Expand and simplify
(x + 2)(x + 1)(x + 3)

Expand the first two brackets: $(x + 2)(x + 1) = x^2 + 3x + 2$
Multiply by the third bracket: $(x^2 + 3x + 2)(x + 3)$
$= x^3 + 3x^2 + 2x + 3x^2 + 9x + 6$
$= x^3 + 6x^2 + 11x + 6$
Answer: $x^3 + 6x^2 + 11x + 6$

5 Reverse fractions

Chloe reads $\frac{4}{9}$ of a book and has now read 40 pages.

How many pages does the whole book have?

$\frac{4}{9}$ of the total = 40

$\frac{1}{9}$ of the total = 40 ÷ 4 = 10

Whole = 10 × 9 = 90
Answer: 90

6 Combined means

The mean mark of 10 adults in a test is 65.
The mean mark of 10 children in the same test is 46.
Work out the mean mark of all 20 people.

Total for adults = 10 × 65 = 650
Total for children = 10 × 46 = 460
Total = 1110, and number of students = 20
Mean = 1110 ÷ 20 = 55.5
Answer: 55.5

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7 Volume of a hollow cylinder

The diagram shows the cross-section of a metal pipe.
The outer radius is 8 cm and the inner radius is 6 cm.
The pipe is 20 cm long.
Work out the volume of metal used. Give your answer in terms of π.

Area of the ring = $\pi \times 8^2 - \pi \times 6^2 = \pi(64 - 36) = 28\pi$
Volume = area × length = $28\pi \times 20$
$= 560\pi$

Answer: 560π cm^3

8 Quadratic graph from its roots

The diagram shows the graph of $y = -x^2 + bx + c$.
Find the value of b and the value of c.

The curve crosses the x-axis at $x = -1$ and $x = 2$
So $y = -(x + 1)(x - 2)$
Expand: $y = -x^2 + x + 2$
$b = 1$ and $c = 2$

Answer: b = 1, c = 2

9 Rationalising with conjugates

Rationalise the denominator of $\frac{4}{3 + \sqrt{5}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $3 - \sqrt{5}$: $\frac{4(3 - \sqrt{5})}{(3 + \sqrt{5})(3 - \sqrt{5})}$

Denominator: $9 - 5 = 4$

$\frac{4(3 - \sqrt{5})}{4} = 1(3 - \sqrt{5})$

$= 3 - \sqrt{5}$

Answer: $3 - \sqrt{5}$

10 Volume of composite solids

A solid is made from a cylinder and a cone.
The cylinder has radius 6 cm and height 6 cm.
The cone has the same radius as the cylinder and perpendicular height 6 cm.
Work out the total volume. Give your answer in terms of π.

Cylinder = $\pi \times 6^2 \times 6 = 216\pi$

Cone = $\frac{1}{3} \times \pi \times 6^2 \times 6 = 72\pi$

Total = $216\pi + 72\pi = 288\pi$

Answer: 288π cm^3