Day 294 **A Tier 10-a-day: Model Solutions**

1 Direct proportion

Here are the ingredients for 4 muffins.
Flour: 120 g
Butter: 100 g
Milk: 80 ml
(a) Work out how much flour is needed for 14 muffins.
(b) Hannah has 633 g of butter. What is the greatest number of muffins she can make?

(a) flour for one: 120 ÷ 4 = 30 g
For 14: 30 × 14 = 420 g
(b) butter for one: 100 ÷ 4 = 25 g
633 ÷ 25 = 25 remainder 8, so she can make 25
Answer: (a) 420 g (b) 25

2 Mixed number addition and subtraction

Work out $4\frac{2}{5} - 1\frac{1}{2}$

Give your answer as a mixed number in its simplest form.

Write as improper fractions: $4\frac{2}{5} = \frac{22}{5}$ and $1\frac{1}{2} = \frac{3}{2}$

Common denominator 10: $\frac{44}{10} - \frac{15}{10}$

$= \frac{29}{10}$

$= 2\frac{9}{10}$

Answer: $\mathbf{2\frac{9}{10}}$

3 Isosceles triangles and exterior angles

LMN is an isosceles triangle with LM = LN.
MNK is a straight line.
Angle LNK = 118°.
Work out the size of angle x.
The diagram is not drawn accurately.

Angles on a straight line add up to 180°
Angle LNM = 180 − 118 = 62°
Base angles are equal, so angle LMN = 62°
x = 180 − 62 − 62 = 56
Answer: x = 56°

4 Converting units

(a) Change 2.5 km^2 to m^2.
(b) Change 72,000 cm^2 to m^2.

(a) 2.5 km^2 × 1,000,000 = 2,500,000 m^2
(b) 72,000 cm^2 ÷ 10 000 = 7.2 m^2
Answer: (a) 2,500,000 m^2 (b) 7.2 m^2

5 Converting area and volume units

Change 500 cm^3 to litres.

1 litre = 1000 cm^3
500 ÷ 1000 = 0.5
Answer: 0.5 litres

Day 294 **A Tier 10-a-day: Model Solutions**

6 Sectors: finding an angle or radius

A pizza slice is a sector of a circle has radius 12 cm and an arc length of 2π cm.
Work out the angle at the centre of the sector.

Arc length = $\frac{\theta}{360} \times 2 \times \pi \times 12$

$2\pi = \frac{\theta}{360} \times 24\pi$

$\theta = 2 \times 360 \div 24$

$\theta = 30$

Answer: 30°

7 Successive percentage changes

The value of a painting is increased by 20%.
The new value is then increased by 5%.
Work out the overall percentage change.

First change: multiplier 1.2. Second change: multiplier 1.05
Overall multiplier = $1.2 \times 1.05 = 1.26$
$1.26 - 1 = 0.26$, so the overall change is 26%
Answer: 26% increase

8 Circles and tangents

The point P(5, 12) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 5^2 + 12^2 = 169$

(b) Gradient of the radius OP = $\frac{12}{5}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div \frac{12}{5} = -\frac{5}{12}$

$y = -\frac{5}{12}x + c$ through (5, 12): $12 = -\frac{5}{12} \times 5 + c$, so $c = \frac{169}{12}$

$y = -\frac{5}{12}x + \frac{169}{12}$

Answer: (a) $x^2 + y^2 = 169$ (b) $y = -\frac{5}{12}x + \frac{169}{12}$

9 Perpendicular bisector

A is the point (2, −1) and B is the point (8, −3).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $(\frac{2+8}{2}, \frac{(-1)+(-3)}{2}) = (5, -2)$

Gradient of AB = $\frac{-2}{6} = -\frac{1}{3}$

Perpendicular gradient = $-1 \div (-\frac{1}{3}) = 3$

$y = 3x + c$ through (5, −2): $-2 = 3 \times 5 + c$, so $c = -17$
$y = 3x - 17$
Answer: $y = 3x - 17$

Day 294 **A Tier 10-a-day: Model Solutions**

10 Surface area of a prism

The diagram shows a triangular prism.
The cross-section is a right-angled triangle.
Work out the total surface area of the prism.
The diagram is not drawn accurately.

Hypotenuse = $\sqrt{7^2 + 24^2}$ = 25 cm

Two triangular ends: $2 \times \frac{1}{2} \times 7 \times 24 = 168$

Three rectangles: $(7 + 24 + 25) \times 19 = 1064$

Total = 168 + 1064 = 1232

Answer: 1232 cm^2