Day 292 **A Tier 10-a-day: Model Solutions**

1 Difference of two squares

(a) Factorise $36x^2 - 1$

(b) Use the same idea to work out $26^2 - 24^2$ without a calculator.

(a) $36x^2 - 1 = (6x)^2 - 1^2$

$(6x - 1)(6x + 1)$

(b) $a^2 - b^2 = (a + b)(a - b)$, so $26^2 - 24^2 = (26 + 24)(26 - 24)$

$= 50 \times 2 = 100$

Answer: (a) $(6x - 1)(6x + 1)$ (b) 100

2 Exchange rates

The exchange rate is £1 = PLN 5.20. (Calculator)

A hotel room costs PLN 1,508 a night.

Work out the cost in pounds.

Divide by the exchange rate: $1508 \div 5.2$

= £290

Answer: £290

3 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.

(a) Work out the value of x.

(b) Work out the size of the angle marked $(2x + 35)°$.

The diagram is not drawn accurately.

(a) Co-interior angles add up to 180°

$2x + 35 + 3x + 30 = 180$

$5x + 65 = 180$

$5x = 115$

$x = 23$

(b) $2 \times 23 + 35 = 81$

Answer: (a) x = 23 (b) 81°

4 Functions

$f(x) = 3x - 1$

$g(x) = x^2 - 4$

Find fg(6)

$g(6) = 6^2 - 4 = 32$

$f(32) = 3 \times 32 - 1 = 95$

Answer: 95

5 Growth and decay

A colony has 4,000 and grows by 6% per day. (Calculator)

Work out the number of bacteria after 6 days.

Give your answer to the nearest whole number.

Multiplier = 1.06

$4{,}000 \times 1.06^6 = 5{,}674.08$

= 5,674 to the nearest whole number

Answer: 5,674

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6 Combining ratios

p : q = 8 : 5
q : r = 1 : 4
(a) Work out the ratio p : q : r
(b) p, q and r add up to 297. Work out the value of r.

(a) Make the q parts equal: lowest common multiple of 5 and 1 is 5
p : q = 8 : 5 and q : r = 5 : 20
p : q : r = 8 : 5 : 20
(b) 8 + 5 + 20 = 33 parts, so 1 part = 297 ÷ 33 = 9
r = 20 × 9 = 180
Answer: (a) 8 : 5 : 20 (b) 180

7 Sectors: finding an angle or radius

A sector of a circle has radius 15 cm and an arc length of 10π cm.
Work out the angle at the centre of the sector.

Arc length = $\frac{\theta}{360} \times 2 \times \pi \times 15$

$10\pi = \frac{\theta}{360} \times 30\pi$

$\theta = 10 \times 360 \div 30$

$\theta = 120$

Answer: 120°

8 Recurring decimals (mixed)

Write $0.54\dot{3}$ as a fraction in its simplest form.

Let x = 0.54343...
Multiply by 10: 10x = 5.4343...
Multiply by 1000: 1000x = 543.4343...
Subtract: 990x = 543 − 5 = 538

$x = \frac{538}{990} = \frac{269}{495}$

Answer: $\frac{269}{495}$

9 Quadratic sequences

Here are the first five terms of a quadratic sequence.
7, 11, 17, 25, 35
(a) Find an expression for the nth term.
(b) Work out the 8th term.

First differences: 4, 6, 8
Second differences: 2, 2, so a = 2 ÷ 2 = 1
Subtract n^2 from each term (1, 4, 9, 16): 6, 7, 8, 9
This is linear: n + 5
nth term = $n^2 + n + 5$
(b) Substitute n = 8: $1 \times 8^2 + 1 \times 8 + 5 = 77$
Answer: (a) $n^2 + n + 5$ (b) 77

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10 Finding an interest rate

Jack invests £2500 in a savings account. (Calculator)
The account pays compound interest at a fixed rate each year.
After 2 years there is £3025 in the account.
Work out the annual rate of interest.

Let the yearly multiplier be m. Then $2500 \times m^2 = 3025$

$m^2 = 3025 \div 2500 = 1.21$

$m = \sqrt{1.21} = 1.1$

Rate = $(1.1 - 1) \times 100 = 10\%$

Answer: 10%