Day 291 **A Tier 10-a-day: Model Solutions**

1 Surd fractions

Show that $\frac{\sqrt{180}-\sqrt{125}}{\sqrt{5}}$ is an integer, and find its value.

$\sqrt{180} = 6\sqrt{5}$ and $\sqrt{125} = 5\sqrt{5}$

Numerator $= 6\sqrt{5} - 5\sqrt{5} = \sqrt{5}$

$\frac{\sqrt{5}}{\sqrt{5}} = 1$

Answer: 1

2 Similar shapes: area and volume

Two similar model boats have heights 6 cm and 15 cm.
The smaller one has a volume of 72 cm^3.
Work out the volume of the larger one.

Length scale factor $= 15 \div 6 = \frac{5}{2}$

Volume scale factor $= (\frac{5}{2})^3 = \frac{125}{8}$

Volume $= 72 \times \frac{125}{8} = 1125$

Answer: 1125 cm^3

3 Speed, distance and time

A train travels at 60 km/h for 2 hours 45 minutes.
Work out the distance travelled.

45 minutes = 0.75 hour, so time = 2.75 hours
Distance = speed × time = 60 × 2.75 = 165 km
Answer: 165 km

4 Expanding and simplifying

Expand and simplify
$(x + 1)^2 - (x - 6)(x + 3)$

$(x + 1)^2 = x^2 + 2x + 1$
$(x - 6)(x + 3) = x^2 - 3x - 18$
Subtract (take care with the signs): $(x^2 + 2x + 1) - (x^2 - 3x - 18)$
$= 5x + 19$
Answer: 5x + 19

5 Ratio and a difference

The ratio of fiction books to non-fiction books in a library shelf is 9 : 11.
There are 28 more non-fiction books than fiction books.
Work out the number of fiction books.

Difference in parts = 11 − 9 = 2
2 parts = 28, so 1 part = 28 ÷ 2 = 14
fiction books: 9 × 14 = 126
Answer: 126

Day 291 **A Tier 10-a-day: Model Solutions**

6 Similar triangles with parallel lines

In triangle ABC, DE is parallel to BC.
D is on AB and E is on AC.
Work out the value of x.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar (DE is parallel to BC)

AE ÷ AC = AD ÷ AB, so AC = $5 \times \frac{13}{5} = 13$

x = EC = 13 − 5 = 8

Answer: x = 8

7 Transformations of graphs

The graph of y = f(x) has a maximum point at (1, −5).
Write down the coordinates of the turning point of the graph of
(a) y = f(−x)
(b) y = f(x − 1)

(a) The graph is reflected in the y-axis, so the x-coordinate changes sign
The maximum point is (−1, −5)
(b) Replacing x by (x − 1) moves the graph 1 unit right
The maximum point is (2, −5)
Answer: (a) (−1, −5) (b) (2, −5)

8 Bearings and distances

A ship leaves a port and travels 10 km on a bearing of 100°. (Calculator)
It then travels 24 km on a bearing of 010°.
(a) Work out how far it is now from the port.
(b) Work out the bearing of its final position from the port. Give your answer to the nearest degree.

The two bearings differ by 90°, so the path makes a right angle

(a) Distance^2 = $10^2 + 24^2$ = 676, so distance = 26 km

(b) The angle at the port between the first leg and the line to the end: $\tan^{-1}(\frac{24}{10}) = 67.4°$

Bearing = 100° − 67.4° = 32.6°
Bearing = 033°
Answer: (a) 26 km (b) 033°

9 Finding a rate of compound change

Nadia invests £12,000 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £17,280.
Work out the annual rate of interest.

Overall multiplier = £17,280 ÷ £12,000 = 1.44

Take the square root: $\sqrt{1.44} = 1.2$
Rate = (1.2 − 1) × 100 = 20%
Answer: 20%

10 Frequency density

The table gives information about 55 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $20 < m \leq 40$.

(a) Missing frequency = 55 − (10 + 3 + 28) = 14
(b) Class width = 40 − 20 = 20
Frequency density = 28 ÷ 20 = 1.4
Answer: (a) 14 (b) 1.4