Day 289 **A Tier 10-a-day: Model Solutions**

1 nth term of a linear sequence

Here are the first four terms of a sequence.
−5, −12, −19, −26
(a) Find an expression for the nth term.
(b) Is −68 a term of this sequence? You must show your working.

The terms change by −7 each time, so the nth term starts with −7n
−5 − (−7) = 2, so the nth term is −7n + 2
(b) −7n + 2 = −68
−7n = −70, so n = 10
n is a whole number, so −68 is a term of the sequence
Answer: (a) −7n + 2 (b) Yes (n = 10)

2 Standard form calculations

Work out $(9.6 \times 10^{-7}) \div (8 \times 10^{3})$
Give your answer in standard form.

Divide the numbers and subtract the powers of 10: 9.6 ÷ 8 = 1.2 and $10^{-7} \div 10^{3} = 10^{-10}$
1.2×10^{-10}
Answer: 1.2×10^{-10}

3 Sequences

Here are the first four terms of a geometric sequence.
1, −3, 9, −27
(a) Write down the next two terms.
(b) Work out the 7th term.

Divide each term by the one before: −3 ÷ 1 = −3, so each term is multiplied by −3
(a) (−27) × (−3) = 81 and 81 × (−3) = −243
(b) Keep multiplying: 7th term = $1 \times (-3)^{6} = 729$
Answer: (a) 81, −243 (b) 729

4 Median from a frequency table

Here are the results for 29 matches.
(a) Find the median.
(b) Work out the range.

(a) The median is the 15th value (29 + 1) ÷ 2 = 15
Cumulative frequencies: 9, 16, 22, 27, 29
The 15th value is 1
(b) Range = 4 − 0 = 4
Answer: (a) 1 (b) 4

5 Functions

f(x) = 4x + 6
$g(x) = x^{2} - 9$
Find fg(4)

$g(4) = 4^{2} - 9 = 7$
f(7) = 4 × 7 + 6 = 34
Answer: 34

6 Index equations

Solve $5^{x} = \frac{1}{25}$

Write the right-hand side as a power of 5: $\frac{1}{25} = 5^{-2}$
So x = −2
Answer: x = −2

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7 Sectors: finding an angle or radius

A sector has radius 12 cm and area 48π cm^2.
Work out the angle at the centre of the sector.

Area = $\frac{\theta}{360} \times \pi \times 12^2$

$48\pi = \frac{\theta}{360} \times 144\pi$

$\theta = 48 \times 360 \div 144$

$\theta = 120$

Answer: 120°

8 Probability without replacement

A bag contains 3 mint and 4 fruit sweets.
Two sweets are taken at random without replacement.
Work out the probability that the two sweets are the same colour.

P(mint, mint) = $\frac{3}{7} \times \frac{2}{6} = \frac{6}{42}$

P(fruit, fruit) = $\frac{4}{7} \times \frac{3}{6} = \frac{12}{42}$

Add: $\frac{6}{42} + \frac{12}{42} = \frac{3}{7}$

Answer: $\frac{3}{7}$

9 Area of shaded regions with circles

A circle of radius 3 cm fits exactly inside a square.
The circle touches all four sides of the square. (Calculator)
Work out the area of the shaded region.
Give your answer to 3 significant figures.

Side of square = 2 × 3 = 6 cm

Square: 6 × 6 = 36

Circle: $\pi \times 3^2 = 9\pi$

Shaded area = $36 - 9\pi = 7.73$

Answer: 7.73 cm^2

10 Direct proportion formulae

y is directly proportional to the cube of x.
When x = 5, y = 375.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 2.

(a) $y = kx^3$. Substitute: $375 = k \times 125$, so $k = 3$

$y = 3x^3$

(b) $y = 3 \times 8 = 24$

Answer: (a) $y = 3x^3$ (b) 24