Day 288 **A Tier 10-a-day: Model Solutions**

1 Primes

(a) Write down all the prime numbers between 60 and 80.
(b) Write 209 as the product of two prime numbers.

(a) A prime number has exactly two factors. Checking each number between 60 and 80 gives: 61, 67, 71, 73, 79
(b) 209 ÷ 11 = 19, and both 11 and 19 are prime.
Answer: (a) 61, 67, 71, 73, 79 (b) 11 × 19

2 Percentage change

Ali buys a vintage radio for £60.
He sells it for £69.60.
Work out his percentage profit.

Profit = £69.60 − £60.00 = £9.60

Percentage profit = $\frac{£9.60}{£60.00}$ × 100 = 16%

Answer: 16%

3 Circle theorems: angle in a semicircle

XY is a diameter of the circle.
Angle YXZ = 29°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle XZY = 90°
Angles in a triangle add up to 180°
x = 180 − 90 − 29 = 61
Answer: x = 61°

4 Double inequalities

(a) Solve $2 < 3x - 1 < 14$
(b) Write down all the integer values of x that satisfy this inequality.

(a) Add 1 to every part: $3 < 3x < 15$
Divide every part by 3: $1 < x < 5$
(b) The integers are 2, 3, 4
Answer: (a) $1 < x < 5$ (b) 2, 3, 4

5 Percentages of percentages

There are 80 passengers on a plane.
25% of the passengers are children.
60% of the children are under 5 years old.
How many passengers are children under 5?

Children = 25% of 80 = 20
Under 5 = 60% of 20 = 12
Answer: 12

6 Quadratic graph from its roots

The diagram shows the graph of $y = -x^2 + bx + c$.
Find the value of b and the value of c.

The curve crosses the x-axis at x = −1 and x = 4
So $y = -(x + 1)(x - 4)$
Expand: $y = -x^2 + 3x + 4$
b = 3 and c = 4
Answer: b = 3, c = 4

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7 Rationalising the denominator

Rationalise the denominator of $\frac{2}{2\sqrt{2}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{2}$: $\frac{2\sqrt{2}}{2 \times 2}$

$= \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$

Answer: $\frac{\sqrt{2}}{2}$

8 Area of a segment

The diagram shows a circle with centre O and radius 15 cm.
Angle AOB = 120°. (Calculator)
Work out the area of the shaded segment.
Give your answer to 3 significant figures.

Area of sector = $\frac{120}{360} \times \pi \times 15^2 = 235.62$

Area of triangle OAB = $\frac{1}{2} \times 15 \times 15 \times \sin 120° = 97.428$

Segment = $235.62 - 97.428 = 138$

Answer: 138 cm^2

9 Addition and multiplication rules

A and B are independent events.
P(A) = 0.4 and P(B) = 0.3.
(a) Work out P(A and B).
(b) Work out P(A or B).

(a) Independent, so multiply: $0.4 \times 0.3 = 0.12$
(b) P(A or B) = P(A) + P(B) − P(A and B) = $0.4 + 0.3 - 0.12 = 0.58$
Answer: (a) 0.12 (b) 0.58

10 Bearings with the cosine rule

Boat P sails 8 miles from a harbour on a bearing of 125°.
Boat Q sails 11 miles from the same harbour on a bearing of 220°. (Calculator)
Work out the distance between the two boats.
Give your answer to 3 significant figures.

The angle between the two paths = $|125° - 220°| = 95°$

$d^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 95°$

$d^2 = 200.34$

$d = 14.2$

Answer: 14.2 miles