Day 285 **A Tier 10-a-day: Model Solutions**

1 HCF and LCM in index form

A = 3 × 7

B = $2^4 \times 3^2 \times 7^4$

Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.
Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): 3 × 7

LCM: for each prime take the higher power: $2^4 \times 3^2 \times 7^4$

Answer: HCF = 3 × 7, LCM = $2^4 \times 3^2 \times 7^4$

2 Surface area of a cuboid

A closed block is a cuboid measuring 10 cm by 11 cm by 2 cm.
Work out the total surface area.

Faces: 10 × 11, 10 × 2 and 11 × 2 each appear twice

2 × (110 + 20 + 22)

= 2 × 152 = 304

Answer: 304 cm^2

3 Exchange rates

The exchange rate is £1 = PLN 5.20. (Calculator)
Jack changes £500 into Polish zloty and spends PLN 2,080.
Jack changes the rest back into pounds at the same rate.
How much does Jack get back?

Amount received: £500 × 5.2 = PLN 2,600

Amount left: PLN 2,600 − PLN 2,080 = PLN 520

In pounds: 520 ÷ 5.2 = £100

Answer: £100

4 Circle theorems: cyclic quadrilaterals

A, B, C and D are points on a circle.
Angle B = (5x − 5)° and angle D = (3x − 7)°.
(a) Work out the value of x.
(b) Work out the size of angle B.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°

5x − 5 + 3x − 7 = 180

8x − 12 = 180

8x = 192

x = 24

(b) 5 × 24 − 5 = 115

Answer: (a) x = 24 (b) 115°

5 Equations with fractions

Solve $\frac{3x - 6}{2} = \frac{5x - 9}{3}$

Multiply both sides by 6

3(3x − 6) = 2(5x − 9)

9x − 18 = 10x − 18

−x = 0

x = 0

Answer: x = 0

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6 Scatter graphs

The scatter graph shows the hours of practice and the marks in a test for some students.
A line of best fit is drawn.
(a) Describe the correlation.
(b) Estimate the marks for a student who practised for 6 hours.

(a) As hours of practice increases, marks in test increases, so there is positive correlation
(b) Draw a line up from 6 on the horizontal axis to the line of best fit, then across to the vertical axis
The line gives a value of 30
Answer: (a) Positive correlation (b) 30

7 Bounds

A plank is 10 mm long and a pipe is 9 mm long.
Each length is correct to the nearest mm.
Work out the upper bound for the difference in their lengths.

To make the difference as large as possible, use the upper bound of the plank and the lower bound of the pipe.
$10.5 - 8.5 = 2$ mm
Answer: 2 mm

8 Equations with brackets

Solve $-5(x - 1) = 5(x - 7)$

Expand the brackets: $-5x + 5 = 5x - 35$
Simplify: $-5x + 5 = 5x - 35$
Collect x terms on one side and numbers on the other: $-10x = -40$
$x = 4$
Answer: $x = 4$

9 Probability with three selections

A bag contains 3 red counters and 6 blue counters.
Three counters are taken at random without replacement.
Work out the probability that all three counters are red.

$$\text{P(all red)} = \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7}$$

$$= \frac{6}{504} = \frac{1}{84}$$

Answer: $\frac{1}{84}$

10 Simultaneous equations (one quadratic)

Solve the simultaneous equations
$x^2 + y^2 = 289$
$y = x + 7$

Substitute $y = x + 7$ into $x^2 + y^2 = 289$
$x^2 + (x + 7)^2 = 289$
$2x^2 + 14x - 240 = 0$
Divide by 2: $x^2 + 7x - 120 = 0$
$(x + 15)(x - 8) = 0$
$x = -15$ or $x = 8$
Use $y = x + 7$ to find y: when $x = -15$, $y = -8$; when $x = 8$, $y = 15$
Answer: $x = -15$, $y = -8$ and $x = 8$, $y = 15$