Day 283 **A Tier 10-a-day: Model Solutions**

1 Loci and constructions

A point P moves so that it is always 6 cm from a fixed point M.
Describe the locus of P.

All points the same distance from M form a circle
Centre M, radius 6 cm
Answer: A circle with centre M and radius 6 cm

2 Speed, distance and time

A cyclist rides 136 km at 16 km/h.
Work out the time taken. Give your answer in hours and minutes.

Time = distance ÷ speed = 136 ÷ 16 = 8.5 hours
8.5 hours = 8 hours 30 minutes
Answer: 8 hours 30 minutes

3 Forming and solving equations

A rectangle has length (3x + 5) cm and width (x + 1) cm.
The perimeter of the rectangle is 44 cm.
(a) Work out the value of x.
(b) Work out the length of the rectangle.

(a) Perimeter = 2 × (length + width), so 2(3x + 5 + x + 1) = 44
3x + 5 + x + 1 = 22 (half of the perimeter)
4x + 6 = 22
4x = 16
x = 4
(b) Length = 3 × 4 + 5 = 17 cm
Answer: (a) x = 4 (b) 17 cm

4 LCM in context

On a stage, a red light flashes every 8 seconds.
A green light flashes every 14 seconds.
They flash together at the start.
After how many seconds will they next flash together?

Find the LCM of 8 and 14.
$8 = 2^3$ and $14 = 2 \times 7$
LCM = 56
They flash together again after 56 seconds.
Answer: 56 seconds

5 Combining ratios

p : q = 5 : 8
q : r = 1 : 4
(a) Work out the ratio p : q : r
(b) p, q and r add up to 315. Work out the value of r.

(a) Make the q parts equal: lowest common multiple of 8 and 1 is 8
p : q = 5 : 8 and q : r = 8 : 32
p : q : r = 5 : 8 : 32
(b) 5 + 8 + 32 = 45 parts, so 1 part = 315 ÷ 45 = 7
r = 32 × 7 = 224
Answer: (a) 5 : 8 : 32 (b) 224

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6 Square and cube numbers

n is the smallest positive integer such that 175 × n is a square number.
Find the value of n.

Write 175 as a product of prime factors: $175 = 5^2 \times 7$
For a square number every prime must have a power that is a multiple of 2.
Extra primes needed: 7 (needs 1 more)
n = 7
Answer: n = 7

7 Simplifying algebraic fractions

Simplify fully

$$\frac{x^2 + 3x - 28}{x^2 - 9x + 20}$$

Factorise the top: (x − 4)(x + 7)
Factorise the bottom: (x − 4)(x − 5)
Cancel the common factor (x − 4)
Answer: $\frac{x + 7}{x - 5}$

8 Surd fractions

Show that $\frac{\sqrt{175} + \sqrt{112}}{\sqrt{7}}$ is an integer, and find its value.

$\sqrt{175} = 5\sqrt{7}$ and $\sqrt{112} = 4\sqrt{7}$
Numerator = $5\sqrt{7} + 4\sqrt{7} = 9\sqrt{7}$
$\frac{9\sqrt{7}}{\sqrt{7}} = 9$
Answer: 9

9 Circle theorems: cyclic quadrilaterals

A, B, C and D are points on a circle.
Angle B = (2x + 15)° and angle D = (4x − 3)°.
(a) Work out the value of x.
(b) Work out the size of angle B.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
2x + 15 + 4x − 3 = 180
6x + 12 = 180
6x = 168
x = 28
(b) 2 × 28 + 15 = 71
Answer: (a) x = 28 (b) 71°

10 Stem and leaf diagrams

The stem and leaf diagram shows the ages of 11 people.
Key: 1 | 1 represents 11 years
(a) Work out the median age.
(b) Work out the interquartile range.

(a) 11 values, so the median is the 6th value
Counting along the leaves gives 27
(b) LQ is the middle of the lower 5 values = 16
UQ is the middle of the upper 5 values = 42
IQR = 42 − 16 = 26
Answer: (a) 27 years (b) 26 years