Day 28 **A Tier 10-a-day: Model Solutions**

1 Roots and powers

(a) Between which two consecutive whole numbers does $\sqrt[3]{70}$ lie?

(b) Work out $4^3 - 5^2$

(a) $4^3 = 64$ and $5^3 = 125$, and 70 is between these.

(b) $4^3 = 64$ and $5^2 = 25$, so $64 - 25 = 39$

Answer: (a) 4 and 5 (b) 39

2 Loci and constructions

Angle XYL is drawn on a piece of paper.

A point P moves so that it is always the same distance from line YX and line YL.

Describe the locus of P.

Points the same distance from two intersecting lines lie on the angle bisector

The line through Y that cuts angle XYL in half

Answer: The bisector of angle XYL

3 Expanding and simplifying

Expand and simplify

$(x - 5)^2 - (x + 4)(x + 3)$

$(x - 5)^2 = x^2 - 10x + 25$

$(x + 4)(x + 3) = x^2 + 7x + 12$

Subtract (take care with the signs): $(x^2 - 10x + 25) - (x^2 + 7x + 12)$

$= -17x + 13$

Answer: $-17x + 13$

4 Density

A stone has a volume of 250 cm^3.

The density of the material is 8.9 g/cm^3.

Work out the mass of the stone.

Mass = density × volume

= 8.9 × 250 = 2225 g

Answer: 2225 g

5 The quadratic formula

Solve $3x^2 - 9x - 3 = 0$

Give your solutions correct to 2 decimal places. (Calculator)

Use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 3$, $b = -9$, $c = -3$

$x = \frac{9 \pm \sqrt{(-9)^2 - 4 \times 3 \times (-3)}}{6}$

$x = \frac{9 \pm \sqrt{117}}{6}$

$x = 3.30$ or $x = -0.30$

Answer: $x = 3.30$ or $x = -0.30$

6 The sine rule

Work out the value of x. (Calculator)

Give your answer to 3 significant figures.

The diagram is not drawn accurately.

$\frac{x}{\sin 51°} = \frac{12}{\sin 71°}$

$x = \frac{12 \times \sin 51°}{\sin 71°}$

$x = 9.86$

Answer: $x = 9.86$ cm

7 Recurring decimals, show that

(a) Show that $0.\dot{2}\dot{6} = \frac{26}{99}$

(b) Hence write $4.\dot{2}\dot{6}$ as a mixed number in its simplest form.

(a) Let x = 0.262626..., so 100x = 26.2626...

100x − x = 26, so 99x = 26 and $x = \frac{26}{99}$

(b) $4.\dot{2}\dot{6} = 4 + \frac{26}{99} = 4\frac{26}{99}$

Answer: (b) $4\frac{26}{99}$

8 Venn diagrams from information

There are 41 students in a class.
24 students do tennis and 21 students do badminton.
6 students do neither.
(a) Work out how many students do both.
(b) One student is chosen at random. Find the probability that they do tennis only.

(a) Students doing at least one = 41 − 6 = 35
Both = 24 + 21 − 35 = 10
(b) tennis only = 24 − 10 = 14

Probability = $\frac{14}{41}$

Answer: (a) 10 (b) $\frac{14}{41}$

9 Vector geometry

OAB is a triangle. →OA = **a** and →OB = **b**.
P is the point on AB such that AP : PB = 1 : 2.
Find →OP in terms of **a** and **b**.

→AB = −**a** + **b**

$\rightarrow AP = \frac{1}{3}\rightarrow AB = \frac{1}{3}(\mathbf{b} - \mathbf{a})$

$\rightarrow OP = \rightarrow OA + \rightarrow AP = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$

$= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

Answer: →OP = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

10 Stratified sampling

A school has 50 students in Year 9, 80 in Year 10 and 130 in Year 11.
A stratified sample of 26 students is to be taken.
Work out how many students should be chosen from Year 10.

Total = 50 + 80 + 130 = 260

Number from Year 10 = $\frac{80}{260} \times 26$

= 8

Answer: 8