Day 279 **A Tier 10-a-day: Model Solutions**

1 Algebraic rearrangement and change of sign

$f(x) = x^3 + x^2 + 2x + 3$
Show that the equation $f(x) = 0$ has a solution between $x = -2$ and $x = -1$.

$f(-2) = -5$
$f(-1) = 1$
There is a change of sign and f is continuous, so there is a solution between $x = -2$ and $x = -1$
Answer: $f(-2) = -5$ and $f(-1) = 1$ (change of sign)

2 Mean from a frequency table

The table shows information about 20 days.
(a) Work out the mean.
(b) Write down the mode.

(a) Total = $0 \times 2 + 1 \times 5 + 2 \times 1 + 3 \times 8 + 4 \times 4 = 47$
Mean = $47 \div 20 = 2.35$
(b) The highest frequency is 8, so the mode is 3
Answer: (a) 2.35 (b) 3

3 Is it a right-angled triangle?

A triangle has sides of length 9 cm, 12 cm and 16 cm.
Is the triangle right-angled?
You must show your working.

Longest side$^2 = 16^2 = 256$
Sum of the squares of the other two sides $= 9^2 + 12^2 = 81 + 144 = 225$
$256 \neq 225$, so the triangle is not right-angled
Answer: No, it is not right-angled

4 Stem and leaf diagrams

The stem and leaf diagram shows the ages of 15 people.
Key: 3 | 1 represents 31 years
(a) Work out the median age.
(b) Work out the interquartile range.

(a) 15 values, so the median is the 8th value
Counting along the leaves gives 46
(b) LQ is the middle of the lower 7 values = 38
UQ is the middle of the upper 7 values = 60
IQR = $60 - 38 = 22$
Answer: (a) 46 years (b) 22 years

5 Error intervals

$t = 4.9$ is rounded to 2 significant figures.
Write down the error interval for t.

The rounding unit is 0.1, so the value could be up to half of this, 0.05, either side.
Lower bound = $4.9 - 0.05 = 4.85$, upper bound = $4.9 + 0.05 = 4.95$
Answer: $4.85 \leq t < 4.95$

6 Average speed

A train travels 180 km at 90 km/h and then a further 180 km at 30 km/h.
Work out the average speed for the whole journey.

Time for first part = $180 \div 90 = 2$ hours
Time for second part = $180 \div 30 = 6$ hours
Total distance = 360 km, total time = 8 hours
Average speed = $360 \div 8 = 45$ km/h
Answer: 45 km/h

Day 279 **A Tier 10-a-day: Model Solutions**

7 Circle theorems: cyclic quadrilaterals

A, B, C and D are points on a circle.
Angle A = $(4x - 5)°$ and angle C = $(3x + 10)°$.
(a) Work out the value of x.
(b) Work out the size of angle A.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
$4x - 5 + 3x + 10 = 180$
$7x + 5 = 180$
$7x = 175$
$x = 25$
(b) $4 \times 25 - 5 = 95$
Answer: (a) x = 25 (b) 95°

8 Completing the square

Solve $x^2 - 4x - 10 = 0$ by completing the square.
Give your answers in surd form.

$x^2 - 4x - 10 = (x - 2)^2 - 14$
$(x - 2)^2 = 14$
$x - 2 = \pm\sqrt{14}$
$x = 2 \pm \sqrt{14}$
Answer: $x = 2 \pm \sqrt{14}$

9 Compound shapes with circles

The diagram shows a shape made from a rectangle and a semicircle.
Work out the area of the shape.
Give your answer in terms of π.
The diagram is not drawn accurately.

Rectangle: $6 \times 5 = 30$
Semicircle: $\frac{1}{2} \times \pi \times 3^2 = \frac{9}{2}\pi$
Total = $30 + \frac{9}{2}\pi$
Answer: $30 + \frac{9}{2}\pi$ cm^2

10 Finding a rate of compound change

Isla invests £15,500 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 3 years the investment is worth £26,784.
Work out the annual rate of interest.

Overall multiplier = £26,784 ÷ £15,500 = 1.728
Take the cube root: $\sqrt[3]{1.728} = 1.2$
Rate = $(1.2 - 1) \times 100 = 20\%$
Answer: 20%