Day 272 **A Tier 10-a-day: Model Solutions**

1 Direct proportion

Here are the ingredients for 12 muffins.
Butter: 180 g
Sugar: 360 g
Water: 300 ml
(a) Work out how much butter is needed for 14 muffins.
(b) Hannah has 854 g of sugar. What is the greatest number of muffins she can make?

(a) butter for one: 180 ÷ 12 = 15 g
For 14: 15 × 14 = 210 g
(b) sugar for one: 360 ÷ 12 = 30 g
854 ÷ 30 = 28 remainder 14, so she can make 28
Answer: (a) 210 g (b) 28

2 Trigonometry: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

The sides given are the opposite (15) and the hypotenuse (17), so use sin

$\sin x = \frac{15}{17}$

$x = \sin^{-1}(\frac{15}{17})$

$x = 61.9°$

Answer: x = 61.9°

3 Rearranging formulae

Make r the subject of $k = \frac{r - 1}{7}$

Multiply both sides by 7: $7k = r - 1$
Add 1 to both sides
$r = 7k + 1$
Answer: r = 7k + 1

4 Bearings

The bearing of M from L is 300°.
Work out the bearing of L from M.
The diagram is not drawn accurately.

The north lines at L and M are parallel
Back bearing = 300° − 180° = 120°
Answer: 120°

5 Two dice

Two fair dice are thrown.
The two scores are multiplied together.
Work out the probability that the product is less than 6.

Outcomes with a product that is less than 6: (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (3, 1), (4, 1), (5, 1)
There are 10 of these out of 36 equally likely outcomes
$\frac{10}{36} = \frac{5}{18}$

Answer: $\frac{5}{18}$

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6 Bounds

A plank is 29 mm long and a pipe is 11 mm long.
Each length is correct to the nearest mm.
Work out the lower bound for the difference in their lengths.

To make the difference as small as possible, use the lower bound of the plank and the upper bound of the pipe.
28.5 − 11.5 = 17 mm
Answer: 17 mm

7 Completing the square

(a) Write $x^2 - 6x + 11$ in the form $(x + a)^2 + b$
(b) Write down the coordinates of the turning point of the curve $y = x^2 - 6x + 11$

(a) Half of −6 is −3, so $(x - 3)^2$
$(x - 3)^2 = x^2 - 6x + 9$, so subtract 9 to get back to 11
So $x^2 - 6x + 11 = (x - 3)^2 + 2$
(b) The turning point is where the bracket is 0, so x = 3 and y = 2
Answer: (a) $(x - 3)^2 + 2$ (b) (3, 2)

8 Similar shapes: area and volume

Two similar bottles have heights 3 cm and 9 cm.
The smaller one has a volume of 3 cm^3.
Work out the volume of the larger one.

Length scale factor = 9 ÷ 3 = 3
Volume scale factor = $(3)^3 = 27$
Volume = 3 × 27 = 81
Answer: 81 cm^3

9 Quadratic inequalities

Solve $x^2 + 7x + 10 < 0$

Solve $x^2 + 7x + 10 = 0$: $(x + 5)(x + 2) = 0$
x = −5 or x = −2
Sketch the U-shaped curve $y = x^2 + 7x + 10$, crossing the x-axis at −5 and −2
The curve is below the x-axis between the roots
−5 < x < −2
Answer: −5 < x < −2

10 Finding a rate of compound change

Zara invests £12,000 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £17,280.
Work out the annual rate of interest.

Overall multiplier = £17,280 ÷ £12,000 = 1.44
Take the square root: $\sqrt{1.44} = 1.2$
Rate = (1.2 − 1) × 100 = 20%
Answer: 20%