Day 271 **A Tier 10-a-day: Model Solutions**

1 Rounding

(a) Write 3868.3 correct to 3 significant figures.
(b) Write 6.3909 correct to 2 decimal places.

(a) Count 3 significant figures and look at the next digit: 3868.3 rounds to 3870
(b) Look at the digit after decimal place 2: 6.3909 rounds to 6.39
Answer: (a) 3870 (b) 6.39

2 Composite functions

f(x) = 3x + 7
g(x) = $x^2 - 2$
Solve fg(x) = 49

fg(x) = $3(x^2 - 2) + 7 = 3x^2 + 1$
$3x^2 + 1 = 49$, so $3x^2 = 48$
$x^2 = 16$, so $x = \pm 4$
Answer: x = 4 or x = −4

3 Successive percentage changes

The number of members of a gym is increased by 5%.
The new number is then decreased by 5%.
Work out the overall percentage change.

First change: multiplier 1.05. Second change: multiplier 0.95
Overall multiplier = 1.05 × 0.95 = 0.9975
0.9975 − 1 = −0.0025, so the overall change is 0.25%
Answer: 0.25% decrease

4 Direct and inverse proportion

y is directly proportional to the cube of x.
When x = 2, y = 64.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 5.

(a) $y = kx^3$. Substitute x = 2 and y = 64: 64 = k × 8, so k = 8
$y = 8x^3$
(b) $y = 8 \times 5^3 = 8 \times 125 = 1000$
Answer: (a) $y = 8x^3$ (b) 1000

5 Volume of a prism

The diagram shows a prism.
The cross-section of the prism is a trapezium.
Work out the volume of the prism.
The diagram is not drawn accurately.

Area of cross-section = $\frac{1}{2} \times (4 + 7) \times 6 = 33$

Volume = area of cross-section × length
= 33 × 9
= 297
Answer: 297 cm^3

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6 Inverse functions

f(x) = 3x − 8

(a) Find $f^{-1}(x)$

(b) Work out $f^{-1}(-2)$

Let y = 3x − 8

Rearrange to make x the subject: 3x = y + 8, so $x = \frac{y + 8}{3}$

Swap y for x: $f^{-1}(x) = \frac{x + 8}{3}$

(b) $f^{-1}(-2) = 2$

Answer: (a) $f^{-1}(x) = \frac{x + 8}{3}$ (b) 2

7 Recurring decimals, show that

(a) Show that $0.\dot{1}\dot{6} = \frac{16}{99}$

(b) Hence write $3.\dot{1}\dot{6}$ as a mixed number in its simplest form.

(a) Let x = 0.161616..., so 100x = 16.1616...

100x − x = 16, so 99x = 16 and $x = \frac{16}{99}$

(b) $3.\dot{1}\dot{6} = 3 + \frac{16}{99} = 3\frac{16}{99}$

Answer: (b) $3\frac{16}{99}$

8 Surface area of cones and spheres

A solid hemisphere has radius 10 cm.
Work out the total surface area of the hemisphere.
Give your answer in terms of π.

Curved surface = $\frac{1}{2} \times 4\pi r^2 = 200\pi$

Flat circular face = $\pi r^2 = 100\pi$
Total = 300π

Answer: 300π cm^2

9 Adding and subtracting in standard form

Work out $(3.75 \times 10^4) - (3.6 \times 10^3)$
Give your answer in standard form.

Write both numbers using 10^3: $3.75 \times 10^4 = 37.5 \times 10^3$
$(37.5 - 3.6) \times 10^3 = 33.9 \times 10^3$
In standard form: 3.39×10^4

Answer: 3.39×10^4

10 Reading a histogram

The histogram shows information about the length of some items.
(a) Work out the number of items with a length in the class 10 to 30 mm.
(b) Work out an estimate for the number with a length greater than 35 mm.

(a) Frequency = frequency density × class width
1 × 20 = 20
(b) 35 to 40: 1.8 × 5 = 9
Class 40 to 50: 2 × 10 = 20
Class 50 to 60: 2.2 × 10 = 22
Total = 9 + 20 + 22 = 51

Answer: (a) 20 (b) 51