Day 270 **A Tier 10-a-day: Model Solutions**

1 Capture-recapture

A scientist wants to estimate the number of frogs in a pond.
She catches 50 frogs, marks each one and returns them.
Later she catches a second sample of 60 frogs. 5 of these are marked.
Work out an estimate for the total number of frogs in a pond.

Proportion marked in the second sample = $\frac{5}{60}$

This is the same as the proportion in the whole population: $\frac{50}{N}$

$\frac{50}{N} = \frac{5}{60}$, so N = 50 × 60 ÷ 5 = 600

Answer: 600

2 Decimal calculations

(a) Work out 0.6 × 0.07
(b) Work out 4.8 ÷ 0.4

(a) 6 × 7 = 42, and there are 3 decimal places in total, so 0.042
(b) Multiply both numbers by 10: 48 ÷ 4 = 12

Answer: (a) 0.042 (b) 12

3 Growth and decay

The population of a town is 40000. (Calculator)
It increases by 12% each year.
Work out the population after 3 years.
Give your answer to the nearest whole number.

Multiplier = 100% + 12% = 1.12

$40000 \times 1.12^3 = 56197.12$

= 56197 to the nearest whole number

Answer: 56197

4 Rearranging to solve quadratics

Solve $6x^2 + 11x = -4$

Move everything to one side: $6x^2 + 11x + 4 = 0$

Factorise: (3x + 4)(2x + 1) = 0

$x = -\frac{4}{3}$ or $x = -\frac{1}{2}$

Answer: $x = -\frac{4}{3}$ or $x = -\frac{1}{2}$

5 Reverse compound change

The value of a house increases by 20% each year. (Calculator)
After 2 years it is worth £46,800.
Work out its value at the start.

Yearly multiplier = 1.2

Overall multiplier = $1.2^2 = 1.44$

Original value = £46,800 ÷ 1.44 = £32,500

Answer: £32,500

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6 Simultaneous equations (one quadratic)

Solve the simultaneous equations

$y = x^2 + 3x - 8$

$y = x + 7$

Set the y values equal: $x^2 + 3x - 8 = x + 7$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5$ or $x = 3$

Substitute into $y = x + 7$: $y = 2$ and $y = 10$

Answer: $x = -5$, $y = 2$ and $x = 3$, $y = 10$

7 Bearings with the cosine rule

Boat P sails 18 miles from a harbour on a bearing of 215°.

Boat Q sails 8 miles from the same harbour on a bearing of 100°. (Calculator)

Work out the distance between the two boats.

Give your answer to 3 significant figures.

The angle between the two paths = $|215° - 100°| = 115°$

$d^2 = 18^2 + 8^2 - 2 \times 18 \times 8 \times \cos 115°$

$d^2 = 509.71$

$d = 22.6$

Answer: 22.6 miles

8 Combined means

The mean mark of 10 adults in a test is 69.

The mean mark of 10 children in the same test is 42.

Work out the mean mark of all 20 people.

Total for adults = $10 \times 69 = 690$

Total for children = $10 \times 42 = 420$

Total = 1110, and number of students = 20

Mean = $1110 \div 20 = 55.5$

Answer: 55.5

9 Recurring decimals (mixed)

Write $0.64\dot{9}$ as a fraction in its simplest form.

Let x = 0.64949...

Multiply by 10: 10x = 6.4949...

Multiply by 1000: 1000x = 649.4949...

Subtract: $990x = 649 - 6 = 643$

$x = \frac{643}{990}$

Answer: $\frac{643}{990}$

10 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.

Angle BAC = 30° and angle ACD = 45°.

Work out the size of angle x (angle AEB).

Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 45°

In triangle ABE: angle BAE = 30° and angle ABE = 45°

$x = 180 - 30 - 45 = 105$

Answer: $x = 105°$