Day 267 **A Tier 10-a-day: Model Solutions**

1 Standard form notation

(a) Write 0.000011 in standard form.
(b) Write 1.25×10^6 as an ordinary number.

(a) Move the decimal point 5 places to get 1.1: 1.1×10^{-5}
(b) Move the decimal point 6 places to the right: 1,250,000
Answer: (a) 1.1×10^{-5} (b) 1,250,000

2 Substitution

Work out the value of $2x^2 + 2x + 6$ when $x = -4$

Substitute $x = -4$: $2 \times (-4)^2 + 2 \times (-4) + 6$
$= 32 - 8 + 6$
$= 30$
Answer: 30

3 Circle theorems: angle in a semicircle

PQ is a diameter of the circle.
Angle QPR = 41°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle PRQ = 90°
Angles in a triangle add up to 180°
$x = 180 - 90 - 41 = 49$
Answer: x = 49°

4 Simplifying surds

(a) Simplify $\sqrt{27}$
(b) Simplify $\sqrt{27} + \sqrt{75}$

(a) $\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$
(b) $\sqrt{75} = 5\sqrt{3}$, so $3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$
Answer: (a) $3\sqrt{3}$ (b) $8\sqrt{3}$

5 Stratified sampling

A school has 225 students in Year 9, 200 in Year 10 and 325 in Year 11.
A stratified sample of 30 students is to be taken.
Work out how many students should be chosen from Year 10.

Total = 225 + 200 + 325 = 750

Number from Year 10 = $\frac{200}{750} \times 30$

= 8
Answer: 8

6 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 64°.
Work out the size of angle x (angle AOB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
The angles in quadrilateral OAPB add up to 360°
$x = 360 - 90 - 90 - 64 = 116$
Answer: x = 116°

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7 Simultaneous equations in context

1 sandwich and 2 drinks cost £20.
3 sandwiches and 3 drinks cost £39.
Work out the cost of one drink.

Let s = cost of one sandwich in £ and d = cost of one drink in £
s + 2d = 20 (1)
3s + 3d = 39 (2)
Multiply (1) by 3 and (2) by 2 to make the d coefficients match
3s + 6d = 60 (1)
6s + 6d = 78 (2)
The d terms have the same sign, so subtract equation (1) from equation (2)
3s = 18, so s = 6
Substitute s = 6 into equation (1) to find d = 7
Answer: £7

8 Probability with three selections

A bag contains 7 mint sweets and 2 toffee sweets.
Three sweets are taken at random without replacement.
Work out the probability that exactly one of the three sweets is toffee.

One order, for example toffee, mint, mint: $\frac{2}{9} \times \frac{7}{8} \times \frac{6}{7} = \frac{1}{6}$

There are 3 possible orders, each with the same probability

$3 \times \frac{1}{6} = \frac{1}{2}$

Answer: $\mathbf{\frac{1}{2}}$

9 Loci: area of a region

A goat is tied by a rope of length 12 m to the corner of a square field of side 16 m.
The goat can graze inside the field only.
Work out the area it can graze. Give your answer in terms of π.
The diagram is not drawn accurately.

The rope can sweep a quarter circle (90°)

Area = $\frac{1}{4} \times \pi \times 12^2$

$= 36\pi$

Answer: 36π m^2

10 Forming quadratic equations

A rectangle has width x cm and length (x + 1) cm.
The area of the rectangle is 20 cm^2.
(a) Show that $x^2 + 1x - 20 = 0$
(b) Work out the value of x.

(a) Area = length × width, so $x(x + 1) = 20$
$x^2 + 1x = 20$
$x^2 + 1x - 20 = 0$
(b) Factorise: $(x - 4)(x + 5) = 0$
$x = 4$ or $x = -5$
A length cannot be negative, so $x = 4$
Answer: (a) Shown (b) x = 4