Day 265 **A Tier 10-a-day: Model Solutions**

1 Standard form notation

(a) Write 0.0023 in standard form.
(b) Write 3.4×10^3 as an ordinary number.

(a) Move the decimal point 3 places to get 2.3: 2.3×10^{-3}
(b) Move the decimal point 3 places to the right: 3,400
Answer: (a) 2.3×10^{-3} (b) 3,400

2 Congruent triangles

Triangle STU: ST = 13 cm, TU = 8 cm, SU = 6 cm.
Triangle LMN: LM = 8 cm, MN = 6 cm, LN = 13 cm.
Are the two triangles congruent?
Give a reason for your answer.

ST = LN = 13 cm
TU = LM = 8 cm
SU = MN = 6 cm
All three pairs of sides are equal, so the triangles are congruent (SSS)
Answer: Yes, congruent (SSS)

3 Two dice

Two fair dice are thrown.
The two scores are added together.
Work out the probability that the sum is 4.

Outcomes with a sum that is 4: (1, 3), (2, 2), (3, 1)
There are 3 of these out of 36 equally likely outcomes
$\frac{3}{36} = \frac{1}{12}$
Answer: $\frac{1}{12}$

4 Equation of a line from a graph

Find the equation of the straight line shown.
Give your answer in the form y = mx + c

The line crosses the y-axis at (0, −3), so c = −3
Gradient = $\frac{\text{change in y}}{\text{change in x}}$: from (0, −3) to (−4, −1), y changes by 2 and x changes by −4
$m = -\frac{1}{2}$
$y = -\frac{1}{2}x - 3$
Answer: $y = -\frac{1}{2}x - 3$

5 Volume of a cylinder

A cylinder has radius 6 cm. It contains water to a depth of 15 cm. (Calculator)
All the water is poured into an empty tank with a rectangular base 15 cm by 20 cm.
Work out the depth of the water in the tank. Give your answer to 3 significant figures.

Volume of water = $\pi \times 6^2 \times 15 = 540\pi$ cm^3
Depth = volume ÷ base area = $540\pi \div (15 \times 20)$
= 5.65
Answer: 5.65 cm

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6 Combining ratios

a : b = 3 : 8
b : c = 4 : 1
(a) Work out the ratio a : b : c
(b) a, b and c add up to 104. Work out the value of c.

(a) Make the b parts equal: lowest common multiple of 8 and 4 is 8
a : b = 3 : 8 and b : c = 8 : 2
a : b : c = 3 : 8 : 2
(b) 3 + 8 + 2 = 13 parts, so 1 part = 104 ÷ 13 = 8
c = 2 × 8 = 16
Answer: (a) 3 : 8 : 2 (b) 16

7 Expanding three brackets

Expand and simplify
(x + 4)(x + 4)(x + 5)

Expand the first two brackets: $(x + 4)(x + 4) = x^2 + 8x + 16$
Multiply by the third bracket: $(x^2 + 8x + 16)(x + 5)$
$= x^3 + 8x^2 + 16x + 5x^2 + 40x + 80$
$= x^3 + 13x^2 + 56x + 80$
Answer: $x^3 + 13x^2 + 56x + 80$

8 Independent events

The probability that Maya wins a game is 0.6.
The probability that Dev wins a game is 0.8.
The two events are independent.
Work out the probability that at least one of them will win a game.

P(neither) = 0.4 × 0.2 = 0.08
P(at least one) = 1 − 0.08 = 0.92
Answer: 0.92

9 Bounds

A rectangle has length 29 mm and width 27 mm.
Each measurement is correct to the nearest mm.
Work out the upper bound for the perimeter of the rectangle.

Upper bound of length = 29.5 mm, upper bound of width = 27.5 mm
Perimeter = 2 × (29.5 + 27.5) = 114 mm
Answer: 114 mm

10 Bearings with the cosine rule

Ship A sails 22 km from a port on a bearing of 155°.
Ship B sails 6 km from the same port on a bearing of 105°. (Calculator)
Work out the distance between the two ships.
Give your answer to 3 significant figures.

The angle between the two paths = |155° − 105°| = 50°
$d^2 = 22^2 + 6^2 - 2 \times 22 \times 6 \times \cos 50°$
$d^2 = 350.30$
d = 18.7
Answer: 18.7 km