Day 264 **A Tier 10-a-day: Model Solutions**

1 Sharing in a ratio

Ravi and Lena share some marbles in the ratio 1 : 6.
Ravi gets 4 marbles.
How many marbles does Lena get?

1 parts = 4, so 1 part = 4 ÷ 1 = 4
Lena gets 6 parts: 6 × 4 = 24
Answer: 24

2 LCM in context

Bus A leaves a bus station every 8 minutes.
Bus B leaves the same station every 20 minutes.
Both buses leave together at 10:05 am.
At what time will they next leave together?

We need the LCM of 8 and 20.
Multiples of 8: 8, 16, 24, 32, 40
Multiples of 20: 20, 40
LCM = 40, so they leave together again 40 minutes later, at 10:45 am
Answer: 10:45 am

3 Frequency density

The table gives information about 87 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $35 < t \leq 55$.

(a) Missing frequency = 87 − (18 + 32 + 13) = 24
(b) Class width = 55 − 35 = 20
Frequency density = 32 ÷ 20 = 1.6
Answer: (a) 24 (b) 1.6

4 Trigonometry: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

The sides given are the opposite (20) and the adjacent (19), so use tan

$\tan x = \frac{20}{19}$

$x = \tan^{-1}(\frac{20}{19})$

$x = 46.5°$

Answer: x = 46.5°

5 Parallel and perpendicular lines

The line L has equation $y = \frac{2}{3}x + 1$

Find the equation of the line that is perpendicular to L and passes through the point $(2, -12)$.

The gradient of L is $\frac{2}{3}$

Perpendicular gradient = $-1 \div \frac{2}{3} = -\frac{3}{2}$

$y = -\frac{3}{2}x + c$, so $-12 = -\frac{3}{2} \times 2 + c$

$c = -9$

$y = -\frac{3}{2}x - 9$

Answer: $y = -\frac{3}{2}x - 9$

Day 264 **A Tier 10-a-day: Model Solutions**

6 Midpoints and length of a line segment

P is the point (3, 2) and Q is the point (−3, 0).
(a) Find the midpoint of PQ.
(b) Work out the length of PQ. Give your answer as a surd in its simplest form.

(a) Midpoint = $(\frac{3+(-3)}{2}, \frac{2+0}{2}) = (0, 1)$

(b) Length = $\sqrt{(-6)^2 + (-2)^2}$
$= \sqrt{40} = 2\sqrt{10}$

Answer: (a) (0, 1) (b) $2\sqrt{10}$

7 Expanding three brackets

Expand and simplify
$(x - 3)(x - 4)(x - 4)$

Expand the first two brackets: $(x - 3)(x - 4) = x^2 - 7x + 12$
Multiply by the third bracket: $(x^2 - 7x + 12)(x - 4)$
$= x^3 - 7x^2 + 12x - 4x^2 + 28x - 48$
$= x^3 - 11x^2 + 40x - 48$

Answer: $x^3 - 11x^2 + 40x - 48$

8 Terminating and recurring decimals

Here are four fractions: $\frac{9}{12}$ $\frac{3}{9}$ $\frac{3}{10}$ $\frac{3}{8}$

Which one of these fractions is equal to a recurring decimal?

Simplify each fraction: $\frac{9}{12} = \frac{3}{4}, \frac{3}{9} = \frac{1}{3}, \frac{3}{10}, \frac{3}{8}$

A fraction in simplest form is a terminating decimal only if its denominator has no prime factors other than 2 and 5.

$\frac{1}{3}$ has denominator 3, which includes a factor other than 2 and 5.

Answer: $\frac{3}{9}$

9 Pythagoras in three dimensions

The diagram shows a cuboid.
Work out the length of the diagonal AG.
The diagram is not drawn accurately.

$AG^2 = 3^2 + 12^2 + 4^2$
$= 9 + 144 + 16 = 169$
$AG = \sqrt{169} = 13$

Answer: 13 cm

10 Bounds and accuracy

A cyclist travels 760 metres in 6.3 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.
(a) Work out the upper bound and the lower bound of the speed in m/s.
(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = 765 ÷ 6.25 = 122.4000
Lower bound = 755 ÷ 6.35 = 118.8976
(b) Both bounds round to 120 to 2 s.f. but not to the same value to 3 s.f.
So the speed is 120 m/s to 2 significant figures.

Answer: (a) 122.4000, 118.8976 (b) 120 m/s (2 s.f.)