Day 261 **A Tier 10-a-day: Model Solutions**

1 Factorising

Factorise fully $8x^2 - 288$

Take out the common factor 8: $8x^2 - 288 = 8(x^2 - 36)$
$x^2 - 36$ is a difference of two squares: $(x - 6)(x + 6)$
$8(x - 6)(x + 6)$
Answer: $8(x - 6)(x + 6)$

2 Is it a right-angled triangle?

A triangle has sides of length 24 cm, 24 cm and 7 cm.
Is the triangle right-angled?
You must show your working.

Longest side^2 = 24^2 = 576
Sum of the squares of the other two sides = $7^2 + 24^2 = 49 + 576 = 625$
$576 \neq 625$, so the triangle is not right-angled
Answer: No, it is not right-angled

3 HCF and LCM

(a) Write 21 and 70 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 21 and 70.

(a) $21 = 3 \times 7$ and $70 = 2 \times 5 \times 7$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: 7
LCM: take the larger power of every prime: $2 \times 3 \times 5 \times 7 = 210$
Answer: (a) $21 = 3 \times 7$, $70 = 2 \times 5 \times 7$ (b) HCF = 7, LCM = 210

4 Column vectors

$\mathbf{a} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$.

Work out $2\mathbf{a} + 3\mathbf{b}$.

$2\mathbf{a} = \begin{pmatrix} 12 \\ -10 \end{pmatrix}$ and $3\mathbf{b} = \begin{pmatrix} 9 \\ -9 \end{pmatrix}$

$2\mathbf{a} + 3\mathbf{b} = \begin{pmatrix} 12 + 9 \\ -10 + (-9) \end{pmatrix}$

$= \begin{pmatrix} 21 \\ -19 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{21} \\ \mathbf{-19} \end{pmatrix}$

5 Equation of a straight line

A straight line passes through the points $(-3, 1)$ and $(0, 13)$.
Find the equation of the line.

Gradient = $\frac{13 - 1}{0 - (-3)} = \frac{12}{3} = 4$
Use $y = mx + c$ with $m = 4$ and the point $(-3, 1)$: $1 = 4 \times (-3) + c$
$c = 13$
$y = 4x + 13$
Answer: $y = 4x + 13$

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6 Mean from a frequency table

Here is some information about 50 students.
(a) Work out the mean.
(b) Write down the mode.

(a) Total = 0 × 1 + 1 × 4 + 2 × 22 + 3 × 17 + 4 × 6 = 123
Mean = 123 ÷ 50 = 2.46
(b) The highest frequency is 22, so the mode is 2
Answer: (a) 2.46 (b) 2

7 Reverse fractions

$\frac{2}{7}$ of the books in a library are fiction.
There are 32 fiction books.
Work out the total number of books in the library.

$\frac{2}{7}$ of the total = 32
$\frac{1}{7}$ of the total = 32 ÷ 2 = 16
Whole = 16 × 7 = 112
Answer: 112

8 Average speed

A van travels for 1 hour at 20 km/h,
then for 1 hour 30 minutes at 65 km/h.
Work out the average speed for the whole journey.

Distance 1 = 20 × 1 = 20 km
Distance 2 = 65 × 1.5 = 97.5 km
Total distance = 117.5 km and total time = 2.5 hours
Average speed = 117.5 ÷ 2.5 = 47 km/h
Answer: 47 km/h

9 Completing the square

Solve $x^2 + 2x - 4 = 0$ by completing the square.
Give your answers in surd form.

$x^2 + 2x - 4 = (x + 1)^2 - 5$
$(x + 1)^2 = 5$
$x + 1 = \pm\sqrt{5}$
$x = -1 \pm \sqrt{5}$
Answer: $x = -1 \pm \sqrt{5}$

10 Probability with three selections

A bag contains 3 black pens and 7 blue pens.
Three pens are taken at random without replacement.
Work out the probability that at least one of the three pens is blue.

P(none is blue) = P(all black) = $\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$

P(at least one blue) = $1 - \frac{1}{120} = \frac{119}{120}$

Answer: $\frac{119}{120}$