Day 26 **A Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

$\frac{3}{5}$ 64% 40% 0.17

Change each to a decimal: $\frac{3}{5}$ = 0.6, 64% = 0.64, 40% = 0.4, 0.17 = 0.17

Order the decimals: 0.17 < 0.4 < 0.6 < 0.64

Answer: 0.17, 40%, $\frac{3}{5}$, 64%

2 Map scales

The scale of a map is 1 : 20000.
Two towns are 1.5 cm apart on the map.
Work out the real distance between the towns in kilometres.

Real distance = 1.5 × 20000 = 30000 cm
30000 cm = 300 m = 0.3 km
Answer: 0.3 km

3 Congruent triangles

Triangle DEF: angle E = 50°, angle F = 45°, EF = 11 cm.
Triangle PQR: angle Q = 45°, angle R = 50°, QR = 11 cm.
Are the two triangles congruent?
Give a reason for your answer.

angle E = angle R = 50°
angle F = angle Q = 45°
EF = QR = 11 cm, the side between the two angles
Two angles and the included side are equal, so the triangles are congruent (ASA)
Answer: Yes, congruent (ASA)

4 Laws of indices

Simplify fully

(a) $\frac{16x^6y^6}{5x^2y^3}$

(b) $(3x^2y^2)^2$

(a) Divide the numbers: 16 ÷ 5 = $\frac{16}{5}$

Subtract the powers: $x^6 \div x^2 = x^4$ and $y^6 \div y^3 = y^3$

= $\frac{16x^4y^3}{5}$

(b) Raise every part to the power 2: $3^2 = 9$, $(x^2)^2 = x^4$, $(y^2)^2 = y^4$

= $9x^4y^4$

Answer: (a) $\frac{16x^4y^3}{5}$ (b) $9x^4y^4$

5 Distance-time graphs

The graph shows the distance travelled by a motorcyclist.
(a) For how many minutes did the motorcyclist stop?
(b) Work out the speed during the final part of the journey. Give your answer in km/h.

(a) The graph is horizontal from 30 to 40 minutes, so the stop is 40 − 30 = 10 minutes
(b) Final part: distance 30 km in 60 minutes

Speed = distance ÷ time = 30 ÷ $\frac{60}{60}$ = 30 km/h

Answer: (a) 10 minutes (b) 30 km/h

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6 Recurring decimals to fractions

Write the recurring decimal $0.\dot{2}\dot{4}$ as a fraction in its simplest form.

Let x = 0.242424...
Multiply by 100: 100x = 24.2424...
Subtract: 99x = 24
$x = \frac{24}{99} = \frac{8}{33}$

Answer: $\frac{8}{33}$

7 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + 4x - 26 = 0$ can be rearranged to give $x = \frac{26 - x^3}{4}$

(b) Show that the equation has a solution between x = 2 and x = 3

(a) Add 26 and subtract 4x from both sides: $4x = 26 - x^3$

Divide both sides by 4: $x = \frac{26 - x^3}{4}$

(b) Let $f(x) = x^3 + 4x - 26$
f(2) = −10
f(3) = 13
There is a change of sign (negative to positive) and f is continuous, so there is a solution between x = 2 and x = 3

Answer: Shown: f(2) = −10, f(3) = 13 (change of sign)

8 Surface area of a prism

The diagram shows a triangular prism.
The cross-section is a right-angled triangle.
Work out the total surface area of the prism.
The diagram is not drawn accurately.

Hypotenuse = $\sqrt{9^2 + 12^2} = 15$ cm

Two triangular ends: $2 \times \frac{1}{2} \times 9 \times 12 = 108$

Three rectangles: (9 + 12 + 15) × 8 = 288
Total = 108 + 288 = 396

Answer: 396 cm²

9 Loci: area of a region

A dog is tied by a rope of length 6 m to the corner of a rectangular shed.
The shed measures 10 m by 9 m.
The dog can graze on any grass it can reach outside the shed.
Work out the area it can graze. Give your answer in terms of π.
The diagram is not drawn accurately.

The rope can sweep a sector of angle 360° − 90° = 270°

Area = $\frac{270}{360} \times \pi \times 6^2$

$= \frac{3}{4} \times 36\pi$

$= 27\pi$

Answer: 27π m²

10 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{5x - 2}{3x + 1}$

Multiply both sides by (3x + 1): $y(3x + 1) = 5x - 2$
Expand the bracket: $3xy + y = 5x - 2$
Collect the x terms on one side: $3xy - 5x = -y - 2$
Factorise: $x(3y - 5) = -y - 2$
Divide by (3y − 5): $x = \frac{-y - 2}{3y - 5}$

Answer: $x = \frac{-y - 2}{3y - 5}$