Day 258 **A Tier 10-a-day: Model Solutions**

1 HCF and LCM

(a) Write 50 and 60 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 50 and 60.

(a) $50 = 2 \times 5^2$ and $60 = 2^2 \times 3 \times 5$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2 \times 5 = 10$
LCM: take the larger power of every prime: $2^2 \times 3 \times 5^2 = 300$
Answer: (a) $50 = 2 \times 5^2$, $60 = 2^2 \times 3 \times 5$ (b) HCF = 10, LCM = 300

2 Area of parallelograms and kites

The diagram shows a kite.
The diagonals are 13 cm and 11 cm and cross at right angles.
Work out the area of the kite.
The diagram is not drawn accurately.

Area $= \frac{1}{2} \times$ (diagonal 1) $\times$ (diagonal 2)
$= \frac{1}{2} \times 13 \times 11$
$= 71.5$
Answer: 71.5 cm^2

3 Solving quadratic equations

Solve $2x^2 - 7x + 5 = 0$

Factorise: $(x - 1)(2x - 5) = 0$
$x - 1 = 0$ or $2x - 5 = 0$
$x = 1$ or $x = \frac{5}{2}$
Answer: $x = 1$ or $x = \frac{5}{2}$

4 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked $(2x + 25)°$.
The diagram is not drawn accurately.

(a) Corresponding angles are equal
$2x + 25 = 3x + 15$
$x = 10$
$x = 10$
(b) $2 \times 10 + 25 = 45$
Answer: (a) x = 10 (b) 45°

5 Direct and inverse proportion

y is inversely proportional to x.
When x = 1, y = 8.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 2.

(a) $y = \frac{k}{x}$. Substitute $x = 1$ and $y = 8$: $8 = \frac{k}{1}$, so $k = 8 \times 1 = 8$
$y = \frac{8}{x}$
(b) $y = \frac{8}{2} = 4$
Answer: (a) $y = \frac{8}{x}$ (b) 4

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6 Successive percentage changes

The population of a village is increased by 5%.
The new population is then increased by 15%.
Work out the overall percentage change.

First change: multiplier 1.05. Second change: multiplier 1.15
Overall multiplier = 1.05 × 1.15 = 1.2075
1.2075 − 1 = 0.2075, so the overall change is 20.75%
Answer: 20.75% increase

7 Rationalising the denominator

Rationalise the denominator of $\frac{18}{\sqrt{20}}$

Give your answer in its simplest form.

Simplify the denominator: $\sqrt{20} = 2\sqrt{5}$

$\frac{18}{\sqrt{20}} = \frac{18}{2\sqrt{5}}$

Multiply top and bottom by $\sqrt{5}$: $\frac{18\sqrt{5}}{2 \times 5}$

$= \frac{9\sqrt{5}}{5}$

Answer: $\frac{9\sqrt{5}}{5}$

8 Cumulative frequency graphs

The cumulative frequency graph shows information about the age of 100 visitors.
Use the graph to find an estimate for
(a) the median,
(b) the number of visitors with a age greater than 40 years.

(a) Median at cumulative frequency 100 ÷ 2 = 50
Read across from 50 to the graph, then down to the axis
Median = 45.5 years
(b) At 40 years the cumulative frequency is 39
100 − 39 = 61
Answer: (a) 45.5 years (b) 61

9 Pythagoras in three dimensions

The diagram shows a cuboid.
Work out the length of the diagonal AG. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$AG^2 = 9^2 + 6^2 + 11^2$
$= 81 + 36 + 121 = 238$
$AG = \sqrt{238} = 15.4$
Answer: 15.4 cm

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10 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{5x - 6}{-x - 5}$

Multiply both sides by $(-x - 5)$: $y(-x - 5) = 5x - 6$
Expand the bracket: $-xy - 5y = 5x - 6$
Collect the x terms on one side: $-xy - 5x = 5y - 6$
Factorise: $x(-y - 5) = 5y - 6$
Divide by $(-y - 5)$: $x = \frac{5y - 6}{-y - 5}$
Multiply the top and bottom by -1: $x = \frac{-5y + 6}{y + 5}$

Answer: $x = \frac{-5y + 6}{y + 5}$