Day 257 **A Tier 10-a-day: Model Solutions**

1 Map scales

The scale of a map is 1 : 20000.
Two towns are 4 cm apart on the map.
Work out the real distance between the towns in kilometres.

Real distance = 4 × 20000 = 80000 cm
80000 cm = 800 m = 0.8 km
Answer: 0.8 km

2 Prime factors

Write 200 as a product of its prime factors. Give your answer in index form.

Divide by prime numbers in turn: 200 ÷ 2 = 100, 100 ÷ 2 = 50, 50 ÷ 2 = 25, 25 ÷ 5 = 5, 5 ÷ 5 = 1
200 = 2 × 2 × 2 × 5 × 5
In index form: $200 = 2^3 \times 5^2$
Answer: $200 = 2^3 \times 5^2$

3 Successive percentage changes

The population of a village is increased by 10%.
The new population is then decreased by 15%.
Work out the overall percentage change.

First change: multiplier 1.1. Second change: multiplier 0.85
Overall multiplier = 1.1 × 0.85 = 0.935
0.935 − 1 = −0.065, so the overall change is 6.5%
Answer: 6.5% decrease

4 Area of a segment

The diagram shows a circle with centre O and radius 5 cm.
Angle AOB = 60°. (Calculator)
Work out the area of the shaded segment.
Give your answer to 3 significant figures.

Area of sector = $\frac{60}{360} \times \pi \times 5^2 = 13.090$

Area of triangle OAB = $\frac{1}{2} \times 5 \times 5 \times \sin 60° = 10.825$

Segment = 13.090 − 10.825 = 2.26
Answer: 2.26 cm^2

5 Powers and roots in standard form

Work out $(4 \times 10^6)^2$
Give your answer in standard form.

$(4 \times 10^6)^2 = 4^2 \times (10^6)^2$
$= 16 \times 10^{12}$
$= 1.6 \times 10^{13}$
Answer: 1.6×10^{13}

6 Similar triangles with parallel lines

In triangle ABC, DE is parallel to BC.
D is on AB and E is on AC.
Work out the value of x.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar (DE is parallel to BC)

Scale factor = AB ÷ AD = 11 ÷ 8 = $\frac{11}{8}$

$x = 8 \times \frac{11}{8} = 11$

Answer: x = 11

Day 257 **A Tier 10-a-day: Model Solutions**

7 Mean from a frequency table

Here is some information about 20 people.
(a) Work out the mean.
(b) Write down the mode.

(a) Total = 1 × 5 + 2 × 6 + 3 × 5 + 4 × 1 + 5 × 3 = 51
Mean = 51 ÷ 20 = 2.55
(b) The highest frequency is 6, so the mode is 2
Answer: (a) 2.55 (b) 2

8 Expanding three brackets

Expand and simplify
$(x + 2)(x - 1)(x + 3)$

Expand the first two brackets: $(x + 2)(x - 1) = x^2 + x - 2$
Multiply by the third bracket: $(x^2 + x - 2)(x + 3)$
$= x^3 + x^2 - 2x + 3x^2 + 3x - 6$
$= x^3 + 4x^2 + x - 6$
Answer: $x^3 + 4x^2 + x - 6$

9 Perpendicular bisector

A is the point (3, 3) and B is the point (1, −1).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $(\frac{3 + 1}{2}, \frac{3 + (-1)}{2}) = (2, 1)$

Gradient of AB = $\frac{-4}{-2} = 2$

Perpendicular gradient = $-1 \div 2 = -\frac{1}{2}$

$y = -\frac{1}{2}x + c$ through (2, 1): $1 = -\frac{1}{2} \times 2 + c$, so $c = 2$

$y = -\frac{1}{2}x + 2$

Answer: $y = -\frac{1}{2}x + 2$

10 Sectors: finding an angle or radius

A sector of a circle has angle 270° at the centre and arc length 24π cm.
Work out the radius of the sector.

Arc length = $\frac{270}{360} \times 2 \times \pi \times r$

$24\pi = \frac{3}{2} \times r \times \pi$

$r = 24 \div \frac{3}{2}$

$r = 16$
Answer: 16 cm