Day 255 **A Tier 10-a-day: Model Solutions**

1 Simplifying surds

(a) Simplify $\sqrt{245}$

(b) Simplify $\sqrt{245} + \sqrt{125}$

(a) $\sqrt{245} = \sqrt{49} \times \sqrt{5} = 7\sqrt{5}$

(b) $\sqrt{125} = 5\sqrt{5}$, so $7\sqrt{5} + 5\sqrt{5} = 12\sqrt{5}$

Answer: (a) $7\sqrt{5}$ (b) $12\sqrt{5}$

2 Loci and constructions

A garden has a straight fence. A tree is planted so that it is always 6 m from the fence.
A point S on a plan is 6 m from the fence, on either side.
Describe the locus of S.

Points a fixed distance from a line, on both sides, form two parallel lines
Each is 6 m from the fence

Answer: Two straight lines parallel to the fence, 6 m on each side

3 Expanding with surds

Expand and simplify $(3 + \sqrt{10})^2$

Give your answer in the form $a + b\sqrt{c}$

$(3 + \sqrt{10})(3 + \sqrt{10}) = 9 + 3\sqrt{10} + 3\sqrt{10} + 10$

Collect: $19 + 6\sqrt{10}$

Answer: $19 + 6\sqrt{10}$

4 Equation of a line from a graph

Work out the equation of the straight line.
Give your answer in the form $y = mx + c$

The line crosses the y-axis at $(0, -1)$, so $c = -1$

Gradient = $\frac{\text{change in y}}{\text{change in x}}$: from $(0, -1)$ to $(-4, 3)$, y changes by 4 and x changes by -4

$m = -1$

$y = -x - 1$

Answer: $y = -x - 1$

5 Growth and decay

A colony has 11,000 and grows by 8% per day. (Calculator)
Work out the number of bacteria after 3 days.
Give your answer to the nearest whole number.

Multiplier = 1.08

$11{,}000 \times 1.08^3 = 13{,}856.83$

= 13,857 to the nearest whole number

Answer: 13,857

6 Surface area of a prism

The diagram shows a triangular prism.
The cross-section is a right-angled triangle.
Work out the total surface area of the prism.
The diagram is not drawn accurately.

Hypotenuse = $\sqrt{6^2 + 8^2} = 10$ cm

Two triangular ends: $2 \times \frac{1}{2} \times 6 \times 8 = 48$

Three rectangles: $(6 + 8 + 10) \times 16 = 384$

Total = $48 + 384 = 432$

Answer: 432 cm^2

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7 Probability without replacement

A bag contains 2 mint and 5 fruit sweets.
Two sweets are taken at random without replacement.
Work out the probability that exactly one of the two sweets is mint.

P(mint then not mint) = $\frac{2}{7} \times \frac{5}{6} = \frac{10}{42}$

P(not mint then mint) = $\frac{5}{7} \times \frac{2}{6} = \frac{10}{42}$

Add: $\frac{10}{42} + \frac{10}{42} = \frac{10}{21}$

Answer: $\mathbf{\frac{10}{21}}$

8 Quadratic inequalities

Solve $x^2 - 17x + 52 \leq 0$

Solve $x^2 - 17x + 52 = 0$: $(x - 4)(x - 13) = 0$
$x = 4$ or $x = 13$
Sketch the U-shaped curve $y = x^2 - 17x + 52$, crossing the x-axis at 4 and 13
The curve is below the x-axis between the roots
$4 \leq x \leq 13$
Answer: $4 \leq x \leq 13$

9 Frequency density

The table gives information about 114 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $35 < l \leq 65$.

(a) Missing frequency = 114 − (12 + 12 + 84) = 6
(b) Class width = 65 − 35 = 30
Frequency density = 84 ÷ 30 = 2.8
Answer: (a) 6 (b) 2.8

10 Direct and inverse proportion

y is directly proportional to the cube of x.
When x = 4, y = 320.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 7.

(a) $y = kx^3$. Substitute x = 4 and y = 320: 320 = k × 64, so k = 5
$y = 5x^3$
(b) $y = 5 \times 7^3 = 5 \times 343 = 1715$
Answer: (a) $y = 5x^3$ (b) 1715