Day 252 **A Tier 10-a-day: Model Solutions**

1 Percentages in context

Isla earns £20000 a year.
She pays no tax on the first £12000.
She pays 25% tax on the rest.
Work out how much tax she pays in a year.

Taxable amount = £20000 − £12000 = £8000
Tax = 25% of £8000 = £2000
Answer: £2000

2 Expanding double brackets

Expand and simplify
$(3x + 3)(2x - 4)$

Multiply every term in the first bracket by every term in the second
$= 6x^2 - 12x + 6x - 12$
$= 6x^2 - 6x - 12$
Answer: $6x^2 - 6x - 12$

3 Surface area of a cuboid

A closed block is a cuboid measuring 12 cm by 11 cm by 2 cm.
Work out the total surface area.

Faces: 12 × 11, 12 × 2 and 11 × 2 each appear twice
2 × (132 + 24 + 22)
= 2 × 178 = 356
Answer: 356 cm^2

4 Stratified sampling

A school has 40 students in Year 9, 50 in Year 10 and 40 in Year 11.
A stratified sample of 13 students is to be taken.
Work out how many students should be chosen from Year 10.

Total = 40 + 50 + 40 = 130
Number from Year 10 = $\frac{50}{130} \times 13$
= 5
Answer: 5

5 Iteration

$x_{n+1} = 5 + \frac{5}{x_n}$
$x_0 = 2$
Work out the values of x_1, x_2 and x_3.
Give your answers correct to 3 decimal places. (Calculator)

Substitute $x_0 = 2$ into the formula, then use each answer in the next step (keep the full calculator display)
$x_1 = 7.5$
$x_2 = 5.66667$
$x_3 = 5.88235$
Answer: $x_1 = 7.500$, $x_2 = 5.667$, $x_3 = 5.882$

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6 Fraction calculations

Work out $(3\frac{2}{5} - 2\frac{1}{2}) \times 2\frac{1}{2}$

Give your answer as a mixed number in its simplest form.

Brackets first, as improper fractions: $\frac{17}{5} - \frac{5}{2} = \frac{9}{10}$

Then multiply: $\frac{9}{10} \times \frac{5}{2} = \frac{9}{4}$

$= 2\frac{1}{4}$

Answer: $2\frac{1}{4}$

7 Bearings and distances

A boat leaves a port and travels 24 miles on a bearing of 100°. (Calculator)
It then travels 18 miles on a bearing of 010°.
(a) Work out how far it is now from the port.
(b) Work out the bearing of its final position from the port. Give your answer to the nearest degree.

The two bearings differ by 90°, so the path makes a right angle

(a) Distance^2 = 24^2 + 18^2 = 900, so distance = 30 miles

(b) The angle at the port between the first leg and the line to the end: $\tan^{-1}(\frac{18}{24}) = 36.9°$

Bearing = 100° − 36.9° = 63.1°
Bearing = 063°
Answer: (a) 30 miles (b) 063°

8 Quadratic sequences

Here are the first five terms of a quadratic sequence.
−5, 1, 13, 31, 55
(a) Find an expression for the nth term.
(b) Work out the 12th term.

First differences: 6, 12, 18
Second differences: 6, 6, so a = 6 ÷ 2 = 3
Subtract $3n^2$ from each term (3, 12, 27, 48): −8, −11, −14, −17
This is linear: −3n − 5
nth term = $3n^2 - 3n - 5$
(b) Substitute n = 12: $3 \times 12^2 - 3 \times 12 - 5 = 391$
Answer: (a) $3n^2 - 3n - 5$ (b) 391

9 Finding a rate of compound change

Zara invests £6,500 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £6,762.60.
Work out the annual rate of interest.

Overall multiplier = £6,762.60 ÷ £6,500 = 1.0404
Take the square root: $\sqrt{1.0404} = 1.02$
Rate = (1.02 − 1) × 100 = 2%
Answer: 2%

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10 Bearings with the cosine rule

Boat P sails 13 miles from a harbour on a bearing of 115°.
Boat Q sails 6 miles from the same harbour on a bearing of 160°. (Calculator)
Work out the distance between the two boats.
Give your answer to 3 significant figures.

The angle between the two paths = $|115° - 160°| = 45°$

$d^2 = 13^2 + 6^2 - 2 \times 13 \times 6 \times \cos 45°$

$d^2 = 94.691$

$d = 9.73$

Answer: 9.73 miles