Day 249 **A Tier 10-a-day: Model Solutions**

1 Roots and powers

(a) Between which two consecutive whole numbers does $\sqrt{99}$ lie?
(b) Work out $\sqrt{49} \times \sqrt[3]{8}$

(a) $9^2 = 81$ and $10^2 = 100$, and 99 is between these.
(b) $\sqrt{49} = 7$ and $\sqrt[3]{8} = 2$, so $7 \times 2 = 14$
Answer: (a) 9 and 10 (b) 14

2 Loci and constructions

AB is a straight line of length 5 cm.
A point P moves so that it is always less than 3 cm from the point A.
Describe the region where P can be.

Less than 3 cm from A means inside the circle
Centre A, radius 3 cm
Answer: Inside a circle with centre A and radius 3 cm

3 Inequalities

Solve $-2x - 9 < 2$

Add 9: $-2x < 11$
Divide by -2. Dividing by a negative number reverses the inequality sign
$x > -\frac{11}{2}$

Answer: $x > -\frac{11}{2}$

4 Similar triangles with parallel lines

In triangle ABC, DE is parallel to BC.
D is on AB and E is on AC.
Work out the value of x.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar (DE is parallel to BC)
Scale factor = AB ÷ AD = 5 ÷ 2 = $\frac{5}{2}$

$x = 10 \times \frac{5}{2} = 25$

Answer: x = 25

5 Rearranging formulae (powers and roots)

Make v the subject of $E = \frac{1}{2}mv^2$

Multiply both sides by 2: $2E = mv^2$
Divide by m: $v^2 = \frac{2E}{m}$
Square root both sides

Answer: $v = \sqrt{\frac{2E}{m}}$

6 Tree diagrams

The tree diagram shows probabilities for two events.
(a) Work out the values of x and y.
(b) Work out the probability of rain and late.

(a) Probabilities on pairs of branches add up to 1.
$x = 1 - 0.1 = 0.9$ and $y = 1 - 0.3 = 0.7$
(b) $0.1 \times 0.3 = 0.03$
Answer: (a) x = 0.9, y = 0.7 (b) 0.03

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7 Rationalising the denominator

Rationalise the denominator of $\frac{9}{\sqrt{20}}$

Give your answer in its simplest form.

Simplify the denominator: $\sqrt{20} = 2\sqrt{5}$

$\frac{9}{\sqrt{20}} = \frac{9}{2\sqrt{5}}$

Multiply top and bottom by $\sqrt{5}$: $\frac{9\sqrt{5}}{2 \times 5}$

$= \frac{9\sqrt{5}}{10}$

Answer: $\mathbf{\frac{9\sqrt{5}}{10}}$

8 Finding a missing value from an average

The mean of the five numbers 21, 2, x, 19, 24 is 16.

Work out the value of x.

Total = 5 × 16 = 80

Known numbers add to 66

x = 80 − 66 = 14

Answer: x = 14

9 HCF and LCM

(a) Write 40 and 28 as products of prime factors.

(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 40 and 28.

(a) $40 = 2^3 \times 5$ and $28 = 2^2 \times 7$

(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2^2 = 4$

LCM: take the larger power of every prime: $2^3 \times 5 \times 7 = 280$

Answer: (a) $40 = 2^3 \times 5$, $28 = 2^2 \times 7$ (b) HCF = 4, LCM = 280

10 Expanding three brackets

Expand and simplify

$(x + 2)(2x + 5)(x - 2)$

Expand the first two brackets: $(x + 2)(2x + 5) = 2x^2 + 9x + 10$

Multiply by the third bracket: $(2x^2 + 9x + 10)(x - 2)$

$= 2x^3 + 9x^2 + 10x - 4x^2 - 18x - 20$

$= 2x^3 + 5x^2 - 8x - 20$

Answer: $2x^3 + 5x^2 - 8x - 20$