Day 243 **A Tier 10-a-day: Model Solutions**

1 Cumulative frequency graphs

The cumulative frequency graph shows information about the length of 60 leaves.
Use the graph to find an estimate for
(a) the median,
(b) the number of leaves with a length greater than 30 cm.

(a) Median at cumulative frequency 60 ÷ 2 = 30
Read across from 30 to the graph, then down to the axis
Median = 25 cm
(b) At 30 cm the cumulative frequency is 34
60 − 34 = 26
Answer: (a) 25 cm (b) 26

2 Bearings

The bearing of Q from P is 205°.
Work out the bearing of P from Q.
The diagram is not drawn accurately.

The north lines at P and Q are parallel
Back bearing = 205° − 180° = 25°
Answer: 025°

3 Equations with brackets

Solve $3(x - 4) = -4(x - 8) + 12$

Expand the brackets: $3x - 12 = -4x + 32 + 12$
Simplify: $3x - 12 = -4x + 44$
Collect x terms on one side and numbers on the other: $7x = 56$
$x = 8$
Answer: x = 8

4 Stratified sampling

A company has 80 employees in Sales, 140 in Admin and 140 in Production.
A stratified sample of 36 employees is to be taken.
Work out how many employees should be chosen from Admin.

Total = 80 + 140 + 140 = 360

Number from Admin = $\frac{140}{360} \times 36$

= 14
Answer: 14

5 Trigonometry in context

A tower stands on horizontal ground. (Calculator)
From a point 50 m from the base of the tower, the angle of elevation of the top is 54°.
Work out the height of the tower. Give your answer to 3 significant figures.

$\tan 54° = \frac{\text{height}}{50}$

height = 50 × tan 54°
= 68.8
Answer: 68.8 m

Day 243 **A Tier 10-a-day: Model Solutions**

6 Dividing fractions in context

A bag contains $3\frac{3}{8}$ kg of flour. Each cake uses $\frac{3}{8}$ kg of flour.

How many cakes can be made?

Divide: $3\frac{3}{8} \div \frac{3}{8}$

As improper fractions and multiplying by the reciprocal: $\frac{27}{8} \times \frac{8}{3}$

= 9

Answer: 9

7 Changing a ratio

A bag contains red counters and blue counters in the ratio 5 : 2.
20 more blue counters are added to the bag.
The ratio of red counters to blue counters is now 5 : 7.
How many counters were in the bag at the start?

Let red = 5x and blue = 2x. After adding: 5x : (2x + 20) = 5 : 7

$\frac{5x}{2x + 20} = \frac{5}{7}$, so $7 \times 5x = 5(2x + 20)$

35x = 10x + 100, so 25x = 100, x = 4

Start: 5x + 2x = 7 × 4 = 28 counters

Answer: 28

8 Forming quadratic equations

A rectangle has width x cm and length (x + 2) cm.

The area of the rectangle is 35 cm^2.

(a) Show that $x^2 + 2x - 35 = 0$

(b) Work out the value of x.

(a) Area = length × width, so x(x + 2) = 35

$x^2 + 2x = 35$

$x^2 + 2x - 35 = 0$

(b) Factorise: (x − 5)(x + 7) = 0

x = 5 or x = −7

A length cannot be negative, so x = 5

Answer: (a) Shown (b) x = 5

9 Square and cube numbers

n is the smallest positive integer such that 675 × n is a square number.
Find the value of n.

Write 675 as a product of prime factors: $675 = 3^3 \times 5^2$

For a square number every prime must have a power that is a multiple of 2.

Extra primes needed: 3 (needs 1 more)

n = 3

Answer: n = 3

Day 243 **A Tier 10-a-day: Model Solutions**

10 Quadratic sequences

Here are the first five terms of a quadratic sequence.

−1, 5, 17, 35, 59

(a) Find an expression for the nth term.

(b) Work out the 10th term.

First differences: 6, 12, 18

Second differences: 6, 6, so a = 6 ÷ 2 = 3

Subtract $3n^2$ from each term (3, 12, 27, 48): −4, −7, −10, −13

This is linear: $-3n - 1$

nth term = $3n^2 - 3n - 1$

(b) Substitute n = 10: $3 \times 10^2 - 3 \times 10 - 1 = 269$

Answer: (a) $3n^2 - 3n - 1$ (b) 269