Day 242 **A Tier 10-a-day: Model Solutions**

1 Sequences

Here are the first four terms of a geometric sequence.
3, 9, 27, 81
(a) Write down the next two terms.
(b) Work out the 8th term.

Divide each term by the one before: 9 ÷ 3 = 3, so each term is multiplied by 3
(a) 81 × 3 = 243 and 243 × 3 = 729
(b) Keep multiplying: 8th term = $3 \times 3^7 = 6561$
Answer: (a) 243, 729 (b) 6561

2 Standard form notation

(a) Write 0.0075 in standard form.
(b) Write 5.07×10^3 as an ordinary number.

(a) Move the decimal point 3 places to get 7.5: 7.5×10^{-3}
(b) Move the decimal point 3 places to the right: 5,070
Answer: (a) 7.5×10^{-3} (b) 5,070

3 Midpoints and length of a line segment

P is the point (0, −6) and Q is the point (6, 2).
(a) Find the midpoint of PQ.

(a) Midpoint = $(\frac{0+6}{2}, \frac{-6+2}{2}) = (3, -2)$
Answer: (a) (3, −2)

4 Roots and powers

(a) Between which two consecutive whole numbers does $\sqrt{43}$ lie?
(b) Work out $\sqrt{36} \times \sqrt[3]{27}$

(a) $6^2 = 36$ and $7^2 = 49$, and 43 is between these.
(b) $\sqrt{36} = 6$ and $\sqrt[3]{27} = 3$, so $6 \times 3 = 18$
Answer: (a) 6 and 7 (b) 18

5 Trigonometry in context

A straight ramp is 6 m long. (Calculator)
It rises through a vertical height of 0.8 m.
Work out the angle the ramp makes with the horizontal. Give your answer to 1 decimal place.

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{0.8}{6}$

$\theta = \sin^{-1}(\frac{0.8}{6})$

$\theta = 7.7°$

Answer: 7.7°

6 Inverse proportion formulae

y is inversely proportional to x^2.
When x = 8, y = 11.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 5.

(a) $y = \frac{k}{x^2}$. Substitute: $11 = \frac{k}{64}$, so $k = 11 \times 64 = 704$

$y = \frac{704}{x^2}$

(b) $y = \frac{704}{25} = 28.16$

Answer: (a) $y = \frac{704}{x^2}$ (b) 28.16

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7 Forming quadratic equations

A rectangle has width x cm and length (x + 7) cm.
The area of the rectangle is 30 cm^2.
(a) Show that $x^2 + 7x - 30 = 0$
(b) Work out the value of x.

(a) Area = length × width, so $x(x + 7) = 30$
$x^2 + 7x = 30$
$x^2 + 7x - 30 = 0$
(b) Factorise: $(x - 3)(x + 10) = 0$
$x = 3$ or $x = -10$
A length cannot be negative, so $x = 3$
Answer: (a) Shown (b) x = 3

8 Reading a histogram

The histogram shows information about the mass of some items.
(a) Work out the number of items with a mass in the class 20 to 40 g.
(b) Work out an estimate for the number with a mass greater than 50 g.

(a) Frequency = frequency density × class width
2.4 × 20 = 48
(b) 50 to 60: 2.8 × 10 = 28
Class 60 to 70: 2 × 10 = 20
Total = 28 + 20 = 48
Answer: (a) 48 (b) 48

9 Probability with three selections

A bag contains 5 mint sweets and 3 toffee sweets.
Three sweets are taken at random without replacement.
Work out the probability that exactly one of the three sweets is toffee.

One order, for example toffee, mint, mint: $\frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} = \frac{5}{28}$
There are 3 possible orders, each with the same probability
$3 \times \frac{5}{28} = \frac{15}{28}$
Answer: $\mathbf{\frac{15}{28}}$

10 Percentages of percentages

In a school, 60% of the students are boys. (Calculator)
40% of the boys and 60% of the girls walk to school.
What percentage of all the students walk to school?

Take 500 students: boys = 300, girls = 200
Boys who walk = 300 × 0.4 = 120
Girls who walk = 200 × 0.6 = 120
Total = 240, so 240 ÷ 500 × 100 = 48%
Answer: 48%