Day 241 **A Tier 10-a-day: Model Solutions**

1 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 78°.
Work out the size of angle x (angle AOB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
The angles in quadrilateral OAPB add up to 360°
x = 360 − 90 − 90 − 78 = 102
Answer: x = 102°

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

57% 0.69 $\frac{1}{4}$ 0.53

Change each to a decimal: 57% = 0.57, 0.69 = 0.69, $\frac{1}{4}$ = 0.25, 0.53 = 0.53

Order the decimals: 0.25 < 0.53 < 0.57 < 0.69

Answer: $\frac{1}{4}$, 0.53, 57%, 0.69

3 Speed, distance and time

A cyclist rides 129 km at 12 km/h.
Work out the time taken. Give your answer in hours and minutes.

Time = distance ÷ speed = 129 ÷ 12 = 10.75 hours
10.75 hours = 10 hours 45 minutes
Answer: 10 hours 45 minutes

4 Expanding double brackets

Expand and simplify
(x − 4)(2x + 2)

Multiply every term in the first bracket by every term in the second
= $2x^2$ + 2x − 8x − 8
= $2x^2$ − 6x − 8
Answer: $2x^2$ − 6x − 8

5 Exact trigonometric values

XYZ is a right-angled triangle with the right angle at Y.
XZ = 12 cm and angle YXZ = 30°.
Work out the exact length of YZ.
Do not use a calculator.

YZ is opposite the 30° angle and XZ is the hypotenuse, so use sin

sin 30° = $\frac{1}{2}$

YZ = 12 × $\frac{1}{2}$ = 6

Answer: 6 cm

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6 Capture-recapture

A scientist wants to estimate the number of frogs in a pond.
She catches 60 frogs, marks each one and returns them.
Later she catches a second sample of 50 frogs. 10 of these are marked.
Work out an estimate for the total number of frogs in a pond.

Proportion marked in the second sample = $\frac{10}{50}$

This is the same as the proportion in the whole population: $\frac{60}{N}$

$\frac{60}{N} = \frac{10}{50}$, so N = 60 × 50 ÷ 10 = 300

Answer: 300

7 Adding algebraic fractions

Write as a single fraction

$\frac{-4}{(x-3)} - \frac{5}{(x+4)}$

Common denominator is $(x-3)(x+4)$

$= \frac{-4(x+4) - 5(x-3)}{(x-3)(x+4)}$

$= \frac{-9x-1}{(x-3)(x+4)}$

Answer: $\mathbf{\frac{-9x-1}{(x-3)(x+4)}}$

8 Reading a histogram

The histogram shows information about the time of some items.
(a) Work out the number of items with a time in the class 30 to 60 minutes.
(b) Work out an estimate for the number with a time greater than 65 minutes.

(a) Frequency = frequency density × class width
1 × 30 = 30
(b) 65 to 70: 2.6 × 5 = 13
Class 70 to 90: 2.4 × 20 = 48
Total = 13 + 48 = 61
Answer: (a) 30 (b) 61

9 Volume of composite solids

A solid is made from a cylinder and a hemisphere.
The cylinder has radius 6 cm and height 10 cm.
The hemisphere has the same radius as the cylinder and sits on top of it.
Work out the total volume. Give your answer in terms of π.

Cylinder = $\pi \times 6^2 \times 10 = 360\pi$

Hemisphere = $\frac{2}{3} \times \pi \times 6^3 = 144\pi$

Total = $360\pi + 144\pi = 504\pi$

Answer: 504π cm^3

10 Finding a rate of compound change

Jack invests £14,500 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £16,912.80.
Work out the annual rate of interest.

Overall multiplier = £16,912.80 ÷ £14,500 = 1.1664

Take the square root: $\sqrt{1.1664} = 1.08$

Rate = (1.08 − 1) × 100 = 8%

Answer: 8%