**Day 240** **A Tier 10-a-day: Model Solutions**

## 1 Midpoints and length of a line segment

A is the point (4, −4) and B is the point (−5, 8).
(a) Find the midpoint of AB.
(b) Work out the length of AB. Give your answer as a whole number.

(a) Midpoint = $(\frac{4 + (-5)}{2}, \frac{-4 + 8}{2}) = (-0.5, 2)$

(b) Length = $\sqrt{(-9)^2 + (12)^2}$
$= \sqrt{225} = 15$

**Answer: (a) (−0.5, 2) (b) 15**

## 2 Converting area and volume units

Change 30000 $cm^2$ to $m^2$.

1 $m^2$ = 10 000 $cm^2$
30000 ÷ 10 000 = 3

**Answer: 3 $m^2$**

## 3 Estimation

By rounding each number to 1 significant figure, estimate the value of
$\frac{6.2 \times 7.5}{0.54}$

Round each number to 1 significant figure: 6, 8, 0.5
$\frac{6 \times 8}{0.5} = 96$

**Answer: 96**

## 4 Simultaneous equations

Solve the simultaneous equations
−3x + 2y = 18
−3x − 2y = 6

The y terms have opposite signs, so add the equations
−6x = 24, so x = −4
Substitute x = −4 into equation (1) to find y = 3

**Answer: x = −4, y = 3**

## 5 The sine rule

Work out the value of x. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$\frac{x}{\sin 73°} = \frac{6}{\sin 52°}$

$x = \frac{6 \times \sin 73°}{\sin 52°}$

x = 7.28

**Answer: x = 7.28 m**

## 6 Estimating the mean from grouped data

The table shows the distances, in km, travelled by 25 lorries.
(a) Write down the class that contains the median.
(b) Work out an estimate for the mean.

(a) The median is in position (25 + 1) ÷ 2 = 13. Cumulative frequencies: 8, 11, 15, 22, 25
The median is in the class 30 < d ≤ 40
(b) Midpoints: 15, 25, 35, 45, 55
15 × 8 + 25 × 3 + 35 × 4 + 45 × 7 + 55 × 3 = 815
Estimate = 815 ÷ 25 = 32.6

**Answer: (a) 30 < d ≤ 40 (b) 32.6**

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## 7 Volume of composite solids

A solid is made from a cylinder and a cone.
The cylinder has radius 9 cm and height 10 cm.
The cone has the same radius as the cylinder and perpendicular height 4 cm.
Work out the total volume. Give your answer in terms of π.

Cylinder = $\pi \times 9^2 \times 10 = 810\pi$

Cone = $\frac{1}{3} \times \pi \times 9^2 \times 4 = 108\pi$

Total = $810\pi + 108\pi = 918\pi$

**Answer: $918\pi$ cm$^3$**

## 8 Reverse compound change

The value of a painting increases by 10% each year. (Calculator)
After 3 years it is worth £15,972.
Work out its value at the start.

Yearly multiplier = 1.1

Overall multiplier = $1.1^3 = 1.331$

Original value = £15,972 ÷ 1.331 = £12,000

**Answer: £12,000**

## 9 HCF and LCM

(a) Write 27 and 99 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 27 and 99.

(a) $27 = 3^3$ and $99 = 3^2 \times 11$

(b) HCF: multiply the prime factors in both lists, taking the smaller power: $3^2 = 9$

LCM: take the larger power of every prime: $3^3 \times 11 = 297$

**Answer: (a) $27 = 3^3$, $99 = 3^2 \times 11$ (b) HCF = 9, LCM = 297**

## 10 Circles and tangents

The point P(12, 5) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 12^2 + 5^2 = 169$

(b) Gradient of the radius OP = $\frac{5}{12}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div \frac{5}{12} = -\frac{12}{5}$

$y = -\frac{12}{5}x + c$ through (12, 5): $5 = -\frac{12}{5} \times 12 + c$, so $c = \frac{169}{5}$

$y = -\frac{12}{5}x + \frac{169}{5}$

**Answer: (a) $x^2 + y^2 = 169$ (b) $y = -\frac{12}{5}x + \frac{169}{5}$**