Day 238 **A Tier 10-a-day: Model Solutions**

1 Estimation

By rounding each number to 1 significant figure, estimate the value of

$$\frac{7.9 \times 2.1}{0.35}$$

Round each number to 1 significant figure: 8, 2, 0.4

$$\frac{8 \times 2}{0.4} = 40$$

Answer: 40

2 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked $(2x + 40)°$.
The diagram is not drawn accurately.

(a) Alternate angles are equal

$2x + 40 = x + 48$

$x = 8$

$x = 8$

(b) $2 \times 8 + 40 = 56$

Answer: (a) x = 8 (b) 56°

3 Composite functions

$f(x) = 3x - 6$

$g(x) = x^2 + 4$

Solve $fg(x) = 114$

$fg(x) = 3(x^2 + 4) - 6 = 3x^2 + 6$

$3x^2 + 6 = 114$, so $3x^2 = 108$

$x^2 = 36$, so $x = \pm 6$

Answer: x = 6 or x = −6

4 Standard form in context

One rice grain has a mass of 4×10^{-3} g.
A sample contains 8×10^7 rice grains.
Work out the total mass of the sample.
Give your answer in standard form.

Total mass = $(4 \times 10^{-3}) \times (8 \times 10^7)$

$4 \times 8 = 32$ and $10^{-3} \times 10^7 = 10^4$

$32 \times 10^4 = 3.2 \times 10^5$

Answer: 3.2×10^5 g

5 Completing the square

Solve $x^2 + 6x + 2 = 0$ by completing the square.
Give your answers in surd form.

$x^2 + 6x + 2 = (x + 3)^2 - 7$

$(x + 3)^2 = 7$

$x + 3 = \pm\sqrt{7}$

$x = -3 \pm \sqrt{7}$

Answer: $x = -3 \pm \sqrt{7}$

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6 Percentages of percentages

There are 120 passengers on a plane.
60% of the passengers are children.
50% of the children are under 5 years old.
How many passengers are children under 5?

Children = 60% of 120 = 72
Under 5 = 50% of 72 = 36
Answer: 36

7 Combined means

The mean of 4 numbers is 56.
Another number is added. The mean of the 5 numbers is now 62.
Work out the number that was added.

Total of the first 4 numbers = 4 × 56 = 224
Total of the 5 numbers = 5 × 62 = 310
Number added = 310 − 224 = 86
Answer: 86

8 Circles and tangents

The point P(15, 8) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 15^2 + 8^2 = 289$

(b) Gradient of the radius OP = $\frac{8}{15}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div \frac{8}{15} = -\frac{15}{8}$

$y = -\frac{15}{8}x + c$ through (15, 8): $8 = -\frac{15}{8} \times 15 + c$, so $c = \frac{289}{8}$

$y = -\frac{15}{8}x + \frac{289}{8}$

Answer: (a) $x^2 + y^2 = 289$ (b) $y = -\frac{15}{8}x + \frac{289}{8}$

9 Best buys

A shop sells pasta in 2 different sizes.
Small bag: 200 g for 55p
Large bag: 250 g for 70p
Which bag is the best value for money? You must show your working.

Small bag: 55p ÷ 2 = 27.5p per 100 g
Large bag: 70p ÷ 2.5 = 28p per 100 g
The small bag has the lowest price per 100 g, so it is the best value
Answer: The small bag

10 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.
Angle BAC = 33° and angle ACD = 49°.
Work out the size of angle x (angle AEB).
Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 49°
In triangle ABE: angle BAE = 33° and angle ABE = 49°
x = 180 − 33 − 49 = 98
Answer: x = 98°