Day 237 **A Tier 10-a-day: Model Solutions**

1 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked (5x + 40)°.
The diagram is not drawn accurately.

(a) Co-interior angles add up to 180°
5x + 40 + 2x + 42 = 180
7x + 82 = 180
7x = 98
x = 14
(b) 5 × 14 + 40 = 110
Answer: (a) x = 14 (b) 110°

2 Percentages in context

A sofa costs £300 plus VAT at 20%.
In a sale, the price including VAT is reduced by 20%.
Work out the sale price of the sofa.

VAT = 20% of £300 = £60, so the price with VAT is £360
Reduction = 20% of £360 = £72
Sale price = £360 − £72 = £288
Answer: £288

3 Straight line graphs

A straight line has equation x + 5y = −2
(a) Find the gradient of the line.
(b) Find the coordinates of the point where the line crosses the x-axis.

(a) Rearrange into y = mx + c: 5y = −x − 2

$y = -\frac{1}{5}x - \frac{2}{5}$

The gradient is $-\frac{1}{5}$

(b) On the x-axis, y = 0: x = −2, so x = −2

Answer: (a) $-\frac{1}{5}$ (b) (−2, 0)

4 Capture-recapture

A scientist wants to estimate the number of deer in a forest.
She catches 100 deer, marks each one and returns them.
Later she catches a second sample of 60 deer. 10 of these are marked.
Work out an estimate for the total number of deer in a forest.

Proportion marked in the second sample = $\frac{10}{60}$

This is the same as the proportion in the whole population: $\frac{100}{N}$

$\frac{100}{N} = \frac{10}{60}$, so N = 100 × 60 ÷ 10 = 600
Answer: 600

5 Index equations

Solve $9^x = \frac{1}{9}$

Write both sides as powers of 3: $9 = 3^2$, so $9^x = 3^{2x}$
$\frac{1}{9} = 3^{-2}$, so 2x = −2
x = −1
Answer: x = −1

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6 Circles and tangents

The point P(−8, 15) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = (-8)^2 + 15^2 = 289$

(b) Gradient of the radius OP = $\frac{15}{-8} = -\frac{15}{8}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div (-\frac{15}{8}) = \frac{8}{15}$

$y = \frac{8}{15}x + c$ through (−8, 15): $15 = \frac{8}{15} \times (-8) + c$, so $c = \frac{289}{15}$

$y = \frac{8}{15}x + \frac{289}{15}$

Answer: (a) $x^2 + y^2 = 289$ (b) $y = \frac{8}{15}x + \frac{289}{15}$

7 Exact trigonometric values

PQR is a right-angled triangle with the right angle at Q.
PQ = 14 cm and angle QPR = 45°.
Work out the exact length of QR.
Do not use a calculator.

QR is opposite and PQ is adjacent to the 45° angle, so use tan
tan 45° = 1
QR = 14 × 1 = 14
Answer: 14 cm

8 Reverse compound change

The value of a house increases by 5% each year. (Calculator)
After 2 years it is worth £44,100.
Work out its value at the start.

Yearly multiplier = 1.05
Overall multiplier = $1.05^2 = 1.1025$
Original value = £44,100 ÷ 1.1025 = £40,000
Answer: £40,000

9 Reading a histogram

The histogram shows information about the time of some items.
(a) Work out the number of items with a time in the class 0 to 20 minutes.
(b) Work out an estimate for the number with a time greater than 25 minutes.

(a) Frequency = frequency density × class width
1.6 × 20 = 32
(b) 25 to 30: 1.2 × 5 = 6
Class 30 to 40: 1.4 × 10 = 14
Class 40 to 50: 2.6 × 10 = 26
Total = 6 + 14 + 26 = 46
Answer: (a) 32 (b) 46

10 Finding a rate of compound change

Hassan invests £6,000 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £6,489.60.
Work out the annual rate of interest.

Overall multiplier = £6,489.60 ÷ £6,000 = 1.0816
Take the square root: $\sqrt{1.0816} = 1.04$
Rate = (1.04 − 1) × 100 = 4%
Answer: 4%