Day 233 **A Tier 10-a-day: Model Solutions**

1 Algebraic formulae in context

A mechanic charges £45 plus £55 for each hour of work.
(a) Write a formula for the total cost, £C, after t hours.
(b) Leo pays £430 in total. Work out the number of hours.

(a) C = 45 + 55t
(b) 45 + 55t = 430
55t = 385
t = 7
Answer: (a) C = 45 + 55t (b) 7

2 Prime factors

Write 432 as a product of prime factors, in index form.

Divide by prime numbers in turn: 432 ÷ 2 = 216, 216 ÷ 2 = 108, 108 ÷ 2 = 54, 54 ÷ 2 = 27, 27 ÷ 3 = 9, 9 ÷ 3 = 3, 3 ÷ 3 = 1
432 = 2 × 2 × 2 × 2 × 3 × 3 × 3
In index form: $432 = 2^4 \times 3^3$
Answer: $432 = 2^4 \times 3^3$

3 Is it a right-angled triangle?

A triangle has sides of length 12 cm, 5 cm and 15 cm.
Is the triangle right-angled?
You must show your working.

Longest side^2 = 15^2 = 225
Sum of the squares of the other two sides = $5^2 + 12^2$ = 25 + 144 = 169
225 ≠ 169, so the triangle is not right-angled
Answer: No, it is not right-angled

4 The quadratic formula

Solve $2x^2 + 9x + 5 = 0$
Give your solutions correct to 2 decimal places. (Calculator)

Use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with a = 2, b = 9, c = 5

$x = \frac{-9 \pm \sqrt{9^2 - 4 \times 2 \times 5}}{4}$

$x = \frac{-9 \pm \sqrt{41}}{4}$

x = −0.65 or x = −3.85
Answer: x = −0.65 or x = −3.85

5 Counting

A cafe menu has 2 starters, 5 main courses and 3 desserts.
(a) How many different three-course meals are possible?
(b) A customer chooses a main course and either a starter or a dessert (not both). How many different meals are possible?

(a) 2 × 5 × 3 = 30
(b) 5 choices of main course × (2 + 3) choices of starter or dessert = 5 × 5 = 25
Answer: (a) 30 (b) 25

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6 Similar shapes: finding a length

The two triangles are similar.
Work out the value of x.
The diagrams are not drawn accurately.

Scale factor from the small triangle to the large triangle = $35 \div 14 = \frac{5}{2}$

$x = 10 \times \frac{5}{2} = 25$

Answer: x = 25 cm

7 Venn diagrams from information

There are 34 students in a class.
14 students do drama and 21 students do choir.
9 students do neither.
(a) Work out how many students do both.
(b) One student is chosen at random. Find the probability that they do drama only.

(a) Students doing at least one = $34 - 9 = 25$
Both = $14 + 21 - 25 = 10$
(b) drama only = $14 - 10 = 4$
Probability = $\frac{4}{34} = \frac{2}{17}$

Answer: (a) 10 (b) $\frac{2}{17}$

8 Inverse functions

$f(x) = \frac{x + 6}{4}$

(a) Find $f^{-1}(x)$
(b) Work out $f^{-1}(6)$

Let $y = \frac{x + 6}{4}$

Multiply by 4: $4y = x + 6$, so $x = 4y - 6$
Swap y for x: $f^{-1}(x) = 4x - 6$
(b) $f^{-1}(6) = 18$

Answer: (a) $f^{-1}(x) = 4x - 6$ (b) 18

9 The cosine rule: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

$\cos x = \frac{10^2 + 17^2 - 16^2}{2 \times 10 \times 17}$

$\cos x = \frac{133}{340}$

$x = \cos^{-1}(0.3912)$
$x = 67.0°$

Answer: x = 67.0°

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10 Finding an interest rate

Maya invests £500 in a savings account. (Calculator)
The account pays compound interest at a fixed rate each year.
After 2 years there is £583.20 in the account.
Work out the annual rate of interest.

Let the yearly multiplier be m. Then $500 \times m^2 = 583.2$

$m^2 = 583.2 \div 500 = 1.1664$

$m = \sqrt{1.1664} = 1.08$

Rate = $(1.08 - 1) \times 100 = 8\%$

Answer: 8%